Volatility in Solar Renewable Energy Certificates: Jumps and Fat Tails

Charles F. Mason* Neil A. Wilmot†

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Abstract

Economies are increasingly adopting renewable energy certificates as a policy mechanism to support the transition away from reliance on fossil fuels. We investigate the price of solar renewable energy certificates (SRECs) in New Jersey, allowing for the potential presence of jumps and time-varying volatility. We find that both features play an important role in the stochastic process describing SREC price returns. We also simulate the implied probability that at least one jump would occur in any given month. These implied probabilities indicate that jumps played a consistently important role in both SRECs and electricity prices. Jumps in SRECs appear to have been particularly noteworthy between late 2011 and early 2013, a period when electricity prices in New Jersey were relatively high. This result hints at the potentially important role of market structure in driving fat tails in price returns.

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*H.A. "Dave" True, Jr. Chair in Petroleum and Natural Gas Economics, Department of Economics, University of Wyoming; bambuzlr@uwyo.edu

†University of Minnesota Duluth; Affiliate Institute on the Environment, University of Minnesota; nwilmot@d.umn.edu

1 Introduction

"Change is inevitable, but transformation is by conscious choice." Amara (2014)

Renewable energy sources form the centerpiece of a strategy aimed at reducing green house gas (GHG) emissions. Indeed, a fundamental transformation of the energy system is likely to be required if mankind is to limit potential future damages from climate change (Nemet, 2019). An important part of this energy transition is likely to entail widespread adoption of renewables in general, and most likely solar power in particular.[1]

Solar power generation produces less GHG emissions than electricity deriving from the combustion of fossil fuels. The Energy Information Administration (*EIA*) forecast that solar power, which only accounted for 3% of US generation in 2020, will grow to 20% by 2050 (U.S. Energy Information Administration, 2021). Indeed, between 2014 and 2020, solar generation from photovoltaic (PV) has increase by 389%, in the US. Such growth has been driven by increases in both the Utility scale generation (475%) and small scale (rooftop) generation has also experienced significant growth (271%).[2]

Numerous policy and regulatory frameworks have been proposed to incentivize the instillation of renewable energy, so as to achieve long term reductions in carbon emissions. Many US states have introduced a renewable portfolio standard (RPS) to

[1] (Nemet, 2019, p. 14) writes: "[m]ajor changes are needed to adapt energy systems in order to ensure that PV [photovoltaic] is a central, rather than peripheral, component of energy systems."

[2] The EIA defines utility-scale solar to be generation of at least 1 MW of capacity, while small-scale solar generation would be less than 1 MW of capacity. For a thorough discussion of the various factors that have contributed to the rise of solar energy see Nemet (2019).

stimulate the production of energy from renewable sources. One policy, that can be used to comply with the RPS mandate, is the use of 'renewable energy certificates' (RECs). These RECs require a tradable certificate be issued to the generator of each MWh of renewable energy, who is then allowed to sell the REC in a market. New Jersey (NJ) was an earlier adopter of this system. Having adopted ambitious future targets for solar generation, NJ instituted a market for solar renewable energy certificates (SRECs), production credits awarded to owners of grid-connected solar PV systems (Coulon et al., 2015).

Like many sources of renewable energy, the lion's share of the cost associated with a solar project is born up front. This raises the potential for slower adoption of solar technology. With large up-front costs, the economic benefits of an investment is substantially driven by the flow of earnings after installation, be they in the form of reduced energy bills (for homeowners) or credits from adopting the new technology such as SRECs (Davis and Owens, 2003). This benefit flow must be compared against the (sunk) cost of investment. When either benefits or costs are uncertain, the problem becomes on of "investment under uncertainty" (Dixit and Pindyck, 1994).

For this this type of problem, the decision-maker may choose at any point in time to invest immediately or to delay investment. If the payoff to the investment can either increase or decrease, as will naturally occur in the presence of uncertainty, delaying allows the decision-maker to reduce the potential for *ex post* regrets after making an investment that falls in value. In this way, delaying the investment can generate an increase in expected payoff; this increase in value is akin to the value associated with a financial option.[3] A key point is that factors that increase the option value would strengthen the incentive to delay (Dixit and Pindyck, 1994; Kellogg, 2014). These factors include changes in the underlying variance of the source of uncertainty, but they also can include features that contribute to fat tails in the "stochastic process". Such fat tails can arise because of time-varying volatility or the presence of unanticipated dramatic changes, sometimes called "jumps".

Our goal in this paper is to formally assess the empirical importance of jumps and time-varying stochasticity in SREC prices. Using maximum likelihood estimation, we evaluate the percentage change in three key energy prices: SRECs, electricity spot prices and natural gas spot prices. We find that allowing for jumps and time-varying volatility provides statistically important improvements over models that do not account for such features, for each of these energy prices. The preferred econometric specification, which allows for both time-varying volatility and jumps, can then be used to estimate the implied probability that at least one jump would occur over a particular time interval (in our application, one day). Upon constructing these jump probabilities, we find that the estimated jump probability for SREC prices is large at a number of points in time, in particular from late 2011 to early 2013 – a period of relatively high electricity prices in New Jersey. Because the possibility of jumps raises the option value of waiting to invest, one conjecture is that tighter markets

[3] Many applications in this genre refer to the investment under uncertainty problem with the moniker "real options".

could be associated with a slower pace of investment – which in this application suggests an additional factor that could retard the transition to solar power.[4] We also discuss the implications of allowing for fat tails in SREC price returns, for example by including jumps; here we undertake a simulation analysis that shows the option value of waiting to invest is larger the larger is: the probability of a jump occurring, the expected value of the jump size, or the variance associated with jump sizes.

2 Literature Review

Increasing interest from state and local policymakers in the development of renewable energy markets has led to the rapid adoption of renewable portfolio standards (RPS). Many of the state RPS laws include a "solar carve-out", which mandates a percentage of power supplied derives from solar generation sources. Hence, RPS creates demand for renewable energy and stimulates investment in renewable energy generation. Krasko and Doris (2013) report that, between 1996 and 2007, more than half of U.S. states had formalized RPS. The authors examine the use of state policy as a tool to support the development of distributed generation photovoltaic (PV) markets. Using cross sectional data, the authors find evidence suggesting that market

[4] At the same time, the implied probability of jumps in the natural gas prices were much smaller. One important distinction between the markets for SRECs on the one hand and natural gas on the other is the thickness of the market: the volume of trades in SRECs was much smaller, and the market much less mature, than the market for natural gas. This contrast raises the possibility that a thinner market, which one might expect to be less intensely competitive, might be tied to a greater tendency for jumps. An alternative explanation is that gas can be stored while electricity can not, suggesting that both electricity prices and solar credits could be more prone to larger and more frequent jumps in price.

supporting policies can be effective at increasing overall PV capacity.

SRECs are financial instruments, created by a state, that are granted to producers of solar power when they generate electricity from solar sources in certain states and under certain conditions (Cohen et al., 2022). SRECs have become an important tool for increasing the proportion of solar energy collected for power generation. However, the nascent nature of this market means that the SREC market experiences significant price volatility. Lee et al. (2017) report eight states in the U.S. have SREC markets, with Massachusetts and New Jersey showing the highest residential average retail prices (2013). The higher expected price makes residential installation more attractive, allowing residents to obtain a future revenue stream based on SREC prices. On the other hand, the high volatility causes uncertainty of profits and discourages the installation of solar PV systems.[5]

Using county level data for 13 Northeastern states, Crago and Chernyakhovskiy (2017) examine the impact of policy incentives for residential PV capacity. These authors report that, while rebates are a highly significant predictor of solar capacity additions, SREC prices were not significant. They further argue that the lack of significance *may be due to the volatility in the prices* during the period studied (emphasis added). Under North Carolina's RPS compliance market, installation of solar power

[5]There is the additional concern that prolific adoption of solar panels could induce a measurable increase in electricity supply, thereby lowering price – and hence the value of the investment in solar panels. Indeed, Mwampashi et al. (2021) argue that widespread adoption of rooftop solar in Australia's National Electricity Market depressed the level of spot prices while increasing price volatility; both of these effects would exert an adverse impact on the value of investment in rooftop solar in the first instance. Jha and Gordon (2021) document a similar price effect, arguing that rooftop solar adoption in Western Australia is sufficiently impactful as to induce utilities to occasionally curtail power deliveries.

has only enabled a few large solar power producers to compete with utility companies to finance, install, and operate solar generating systems (Gaul and Carley, 2012).

The potential that heightened price volatility can reduce investment is well-documented in the energy literature. Pindyck (2004) examines the behavior of volatility in both the natural gas and oil markets, arguing that volatility alters the incentives to invest in production facility, storage, and transport infrastructure. Indeed, volatility is a key element for the pricing of derivatives, hedging strategies and the decision to invest in physical capital (real options). Relatedly, Efimova and Serletis (2014) show that GARCH models explain price volatility for crude oil, natural gas and electricity. Mason and Wilmot (2016a) show that delaying investment in physical assets could occur because of discontinuities – or "jumps" – in the relevant market price. Similarly, in an analysis of the market for renewable identification numbers (RINs), Mason and Wilmot (2016b) show that a stochastic model which includes jumps provides a compelling explanation of the data. Their results indicate the statistical importance of incorporating jumps in a model of price returns.

The empirical importance of jumps has been documented for a variety of energy markets, including oil (Askari and Krichene, 2008; Gronwald, 2012; Postali and Picchetti, 2006; Wilmot and Mason, 2013), natural gas (Benth et al., 2008; Mason and Wilmot, 2016a), carbon permits in the European Union trading market (Alberola et al., 2008; Chevallier and Sévi, 2014; Daskalakis et al., 2009; Hammoudeh et al., 2014), and coal (Wilmot, 2016; Xiaoming et al., 2012).[6] A natural consequence of

[6] Indeed, Chevallier and Ielpo (2014) argue that many energy prices are subject to these effects.

jumps in energy commodity prices is that electricity spot prices can also exhibit jumps (Benth et al., 2008; Huisman and Mahieu, 2003). The jumps in these energy prices can be large: Huisman and Mahieu (2003) show that daily jumps in electricity prices can be on the order of 30%. Similarly, Thompson et al. (2009, p. 227) observe "high [natural gas] price spikes far outside normal seasonal equilibrium levels;" and Chen and Forsyth (2010, p. 359) argue it "is not uncommon to see spot gas price jumps ... as much as 20% in a single day."

Similar to this earlier literature, there is evidence that the SREC market in NJ has exhibited volatile price dynamics (Coulon et al., 2015); this has the potential to reduce investments in solar power generation due to the increased risk from large price swings. These authors propose a stochastic model to examine the behavior of SREC market prices, which includes a feedback mechanism for power generation decisions. Utilizing monthly data, over the period July 2008 through March 2014, they model the historical dynamics of SREC prices. The model requires numerical solution algorithm, which the authors argue must be carefully chosen, as the historical SREC price behavior is extremely challenging to capture or understand in a classical econometric price model.

Understanding the nature of these price movements has important implications

These energy markets can sometimes be correlated Gonzalez-Pedraz et al. (2014); Asche et al. (2006) and Panagiotidis and Rutledge (2007) demonstrate a link between UK natural gas prices and European crude prices. On the other hand, (Brown and Yücel, 2008, p. 48) argue that there has been a fundamental change in the relationship between oil and gas prices after 2000. And while the data from 2000 – 2006 is reflective of emerging technological trends, it predates the major burst in gas production that followed the widespread application of fracking. With prices subject to jumps, it is not unexpected that related derivatives would also exhibit jumps – for example, the convenience yield associated with holding crude oil stockpiles (Mason and Wilmot, 2020).

for large-scale investment decisions, particularly when these decisions entail up-front costs that are costly (or impossible) to reverse. Here too there is a long tradition in the literature of analyzing investment decisions when key ingredients, such as underlying prices, are stochastic. Much of this literature follows the seminal work of Dixit and Pindyck (1994). A key insight here is that there is a value to waiting – reflecting the potential for the gain from investing to increase over time; this value is referred to as the "option value" of delaying investment. In general, the larger is the option value from delaying, the later will the investment be undertaken. Of direct relevance to our analysis, Torani et al. (2016) evaluate a model of investment in solar projects under uncertainty. Their model assumes solar installation prices follow geometric Brownian motion; they allow for neither GARCH-style volatility nor jumps in prices.[7] By contrast, we do allow for both GARCH and jumps in our model of SREC prices, and find that both aspects are statistically important.

These results have clear implications for a broad range of decisions involving renewable energy, from larger decisions (such as a utility's decision to install a substantial solar farm) to smaller decisions (such as a homeowner's decision to install rooftop solar). There is a robust literature that has studied problems of investment under uncertainty; many of these papers study applications to energy and climate economics. A number of authors have studied applications to non-renewable energy resources, such as oil or gas, often with a focus on production from a single well. A repre-

[7] They also argue that electricity price uncertainty plays an important role in impacting the volatility of solar markets.

sentative sample of these papers includes Almansour and Insley (2017); Conrad and Kotani (2005); Cortazar and Casassus (1998); Davis and Lund (2018); Insley (2017); Kellogg (2014); Mason (2001); Muehlenbachs (2015) and Paddock et al. (1988). The broad consensus from this literature is that the development of energy resources can often be characterized in a fashion that is consistent with real option theory. In particular, the notion of the option value of waiting, which is central to our discussion, is empirically relevant in these applications.

3 Methods and Materials

3.1 Econometric Methods

To present the stochastic process under investigation, let P_t denote the price of *SREC* at time t, which follows a geometric Browniam motion *(GBM)* process. Define α as the trend, and σ as the variance, so that

$$dP_t = \alpha P_t d_t + \sigma P_t dz. \tag{1}$$

In Eq. 1, dz represents an increment of a Wiener process, $dz = \xi\sqrt{dt}$, where ξ has zero mean and a standard deviation equal to 1. (Dixit and Pindyck (1994)). Let x_t denote the natural logarithm of the ratio of price in period t to the price in period $t-1$, $x_t = ln(P_t) - ln(P_{t-1})$. If P_t follows a *GBM* process then x_t is normally distributed with variance σ^2 and mean $\mu = \alpha - \sigma^2/2$. This gives the pure diffusion *(PD)* model

$$x_t = \mu + \sigma z_t \tag{2}$$

The term z_t in Eq. 2 is an identically and independently distributed (i.i.d.) random variable with mean zero and variance one.

As we observed in Section 2 above, many authors have argued that continuous stochastic processes, such as *GBM*, are insufficient for explaining discontinuous movements, or "jumps", in prices. To introduce jumps into the above model, we adapt the methodology in Merton (1976) – wherein returns are composed of two types of changes. The first, 'normal' fluctuations, are represented through the geometric Brownian motion process, while the second – 'abnormal' shocks due to the arrival of new information – are modeled through a discontinuous process. The discontinuities are described by the Poisson distribution governing the number of discrete-valued events, $\eta_t \in \{0, 1, 2, ...\}$, that occur over the interval $(t-1, t)$,

$$P(N_t = j) = \frac{\exp(-\lambda)\lambda^j}{j!} \tag{3}$$

Eq. 3 is characterized by the jump intensity, λ, which describes the mean number of 'events' per unit of time and is expressed as

$$dn_t = \begin{cases} 0 & \text{with probability } 1 - \lambda dt \\ 1 & \text{with probability } \lambda dt \end{cases} \tag{4}$$

As in Askari and Krichene (2008), when abnormal information arrives at time t, prices jump from P_{t-} (the limit as the time index tends towards t from left) to $P_t = \exp(J_t) P_{t-}$. Accordingly, J_t measures the percentage change in price. The resultant stochastic process for the random variable P_t may then be written as

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dz_t + (\exp(J_t) - 1) dn_t, \tag{5}$$

where dz_t has the same properties assumed in equation (1) and dn_t is the independent Poisson process described in equation (4). Together the terms dz_t and dn_t make up the instantaneous component of the unanticipated return. It is natural to assume these terms are independent, since the first component reflects ordinary movements in price while the second component reflects unusual changes in price. The size of the jump, $Y_{t,k}$, is itself a random variable; we assume it is normally distributed with mean θ and variance δ^2, and that it is independent of the distribution for the arrival of a jump. The jump component affecting returns between time t and time *t+1* is then

$$J_t = \sum_{k=0}^{n_t} Y_{t,k}. \tag{6}$$

Thus, the mixed jump-diffusion (JD) process for the log-price returns can be described by

$$x_t = \mu + \sigma z_t + J_t. \tag{7}$$

The probability density function governing x can be derived by applying Bayes' law (Chan and Maheu, 2002; Maheu and McCurdy, 2004). To this end let $f(x_t|n_t = j, \Omega_{t-1})$ denote the conditional density of returns if j jumps have occurred and given the available information Ω_{t-1}. Based on Bayes rule, when x_t is observed, the posterior probability that j jumps will occur at time t is

$$P(n_t = j|\Omega_{t-1}) = \frac{f(x_t|n_t = j, \Omega_{t-1})P(n_t = j|\Omega_{t-1})}{P(x_t|\Omega_{t-1})}. \tag{8}$$

Then, assuming that the conditional density of x_t is normally distributed, and using equation (3) to describe the probability that j jumps occur, we obtain:

$$f(x_t|n_t = j, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma^2 + j\delta^2)}} \exp(-\frac{(x_t - \mu - \theta j + \theta\lambda)^2}{2(\sigma^2 + j\delta^2)}). \tag{9}$$

Finally, integrating out the discrete valued number of jumps yields an expression for the conditional density in terms of observable variables:

$$P(x_t|\Omega_{t-1}) = \sum_{j=0}^{\infty} f(x_t|n_t = j, \Omega_{t-1})P(n_t = j|\Omega_{t-1}). \tag{10}$$

An alternative explanation for the "fat tails" that are often observed in commodity price data is that P_t is subject to time-varying volatility. An example of such a phenomenon is the "generalized autoregressive conditional heteroskedastic" (GARCH) framework. Adapting the pure diffusion model to allow for this form of time-varying volatility gives the GARCH–diffusion (GPD) process:

$$x_t = \mu + \sqrt{h_t} z_t, \tag{11}$$

where the conditional variance, h_t is described by the process

$$h_t = \kappa + \alpha_1 \left(x_{t-1} - \mu\right)^2 + \beta_1 h_{t-1}. \tag{12}$$

Note that when $h_t = \sigma^2$ the GARCH–diffusion model *(GPD)* reduces to pure diffusion model *(PD)*. On the other hand, when $\kappa > 0$ and $\alpha_1 + \beta_1 < 1$, the unconditional variance of the volatility of the process exists and equals $\frac{\kappa}{1-\alpha_1-\beta_1}$. The process described in equations (11) – (12) is characterized by four parameters, μ, κ, α_1 and β_1. There is a general consensus in the literature is that a GARCH model with a limited number of terms performs reasonably well (Akgiray, 1989; Hansen and Lunde, 2005; Sadorsky, 2006), and so we focus our analysis on this more parsimonious representation.[8] Allowing for jump discontinuities would result in the GARCH(1,1) jump-diffusion (GJD) process:

$$x_t = \mu + \sqrt{h_t} z_t + J_t, \tag{13}$$

[8] Duan (1997) shows that the diffusion limit of a large class of GARCH(1,1) models contain many diffusion processes allowing the approximation of stochastic volatility models by the GARCH process.

where h_t is described by equation (12).

We evaluate the four models using maximum likelihood estimation methods, which are known to provide consistent and invariant estimates, with asymptotically normal distributions of the parameters. To this end, we note that the parameters of our four candidate models – *PD, JD, GPD, GJD* – may be nested into the general log-likelihood function

$$L(\phi, x_t) = -T\lambda + \sum_{t=1}^{T} \ln \left[\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \frac{1}{\sqrt{h_t + n\delta^2}} \exp\left(\frac{-(x_t - \mu - n\theta)}{2(h_t + n\delta^2)} \right) \right] + K, \qquad (14)$$

where K is the constant $\frac{T\ln(2\pi)}{2}$, n indexes the number of jumps, combined with the description of h_t given in equation (12).[9] In this framework, the GPD model corresponds to the parameter restriction $\lambda = \theta = \delta = 0$; the JD model corresponds to the restriction $\alpha_1 = \beta_1 = 0$; and the PD model corresponds to the restriction $\alpha_1 = \beta_1 = \lambda = \theta = \delta = 0$.

Comparing any pair of potential models can thus be framed as a test of an appropriate parameter restriction. For example, the comparison of the PD and GPD models is conducted by testing the parameter restriction $\alpha_1 = \beta_1 = 0$; the comparison of the PD and JD models is conducted by testing the parameter restriction $\lambda = \theta = \delta = 0$. The empirical validity of the parameter restriction of interest can be evaluated by use of the likelihood ratio test. This approach compares the likelihood

[9] To faciliate estimation based on eq. (14), the number of jumps was truncated at 10 (Ball and Torous, 1985). Allowing more jumps did not significantly change the point estimates.

function under a particular restriction, $L(\phi^R;x)$, to that of the unrestricted or less restricted likelihood function, $L\left(\hat{\phi};x\right)$. Under the null hypothesis that the restriction is empirically valid, the decrease in the likelihood function associated with the restriction will be small. Such an approach can be used to make pairwise-comparisons between a more general model and a more restricted model. The test statistic is the log-likelihood ratio

$$LR = 2[L(\hat{\phi};x) - L(\phi^R;x)];$$

under the null hypothesis this statistic will be distributed as a Chi-square random variable with m degrees of freedom, where m is the number of parameter restrictions.

3.2 Data

Our empirical analysis is based on daily price observations. We obtained SREC prices from the New Jersey Clean Energy program website.[10] Natural gas prices are based on the Henry Hub spot price.[11] For electricity, we used the PJM Western hub price[12], as this range includes New Jersey.

Table 1 provides summary statistics for the log-returns, calculated as $x_t = ln(P_t) - ln(P_{t-1})$, for each of the prices returns series. The statistics indicate a significant amount of variation among the three variables, though SERC returns exhibit the lowest relative level of dispersion. Evidence of asymmetry in the distribution of

[10] Accessed at https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive

[11] https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm

[12] https://www.eia.gov/electricity/wholesale/

returns is also highest for SERC returns, while electricity prices exhibit negative skewness. The "fat tails" are evident from the large values of kurtosis. The normality hypothesis of the Kolmogorov-Smirnov test is rejected, for each time-series. Lastly, Table 1 contains the results of the modified ADF test of nonstationarity. In each of the log return series the null hypothesis of a unit root is rejected, in favor of stationarity.[13]

These results are corroborated by Figures 1 and 2. Figure 1 displays the log returns for the three energy series. Notably, large sudden changes in the returns are evident, and appear with less frequency for natural gas returns, though on a larger scale for SREC solar price returns. Also, the large changes in SREC solar price returns are evident, and the positive shocks appear much larger in amplitude than the negative changes. Figure 2 displays "quantile-quantile" plots, for the three return series. This plot compares the quantiles of the empirical distribution (measured on the y-axis) against the quantiles of a theoretical normal distribution (measured on the x-axis). If the empirical distribution is close to a normal distribution, the relation will be well described by a straight line. As demonstrated in the figure, there are significant departures from a linear relation, indicating that the date is not well-described by a normal distribution. Indeed, the significant departures observed in along the ends of the plots are indicative of significant leptokurtosis and cast doubt on the assumptions

[13] While not reported in the table, the SREC solar credit, natural gas, and electricity prices are nonstationary (in levels), and thus first differencing (e.g. log returns) renders the series stationary. The findings are supported by traditional unit root tests, the results of which are available upon request.

of an underlying GBM process.

4 Results

4.1 Maximum Likelihood Estimation

Using maximum likelihood methods, we estimate the parameters for the four stochastic processes (*PD, JD, GPD, GJD*), using the daily returns series. The results of the estimation are presented in Table 2, while the likelihood ratio test statistics, for the pairwise comparison of the four models, are presented in Table 3.

Examining the pure diffusion results, the estimated drift parameter, μ, is positively and statistically significant for SREC returns, though not statistically significant for either natural gas or electricity prices.[14] For each series, the estimated instantaneous rate of variance, σ, is statistically significant. This same pattern holds when examining the drift and variance of the JD models, though allowing for jumps markedly reduces the estimated instantaneous rate of variance. The estimated jump intensity, λ, is statistically significant for each of the three series. The estimated value of λ is largest for the SREC series; this estimate suggests jumps occur quite frequently for SREC returns – roughly two out of every three days. Finally, the estimated mean jump size, θ, is positive when the estimate is significant.

Comparing the results of the PD and GPD models, we see that allowing for

[14] To improve convergence of the estimation procedure, we rescaled the price return series for natural gas and electricity by multiplying by 100; the SREC returns were not rescaled.

GARCH also improved the models predictive power: the log-likelihood function shows improvement, in each of the three series. The estimated coefficients associated with the GARCH model, κ, α_1 and β_1, are statistically significant across each of the three series. Additionally, all three series demonstrate persistent volatility, with $\alpha_1+\beta_1$ close to 1. Notably, the magnitude of the estimated value of κ is large for electricity price returns – pointing to large overall variation in that series. Importantly, our results indicate that combining GARCH with the jump model (GJD) yield the best results, demonstrating the importance of including jumps. We note that the probability of a jump occurring has fallen across all three series, while the GARCH terms remain statistically significant. Evidently, the inclusion of GARCH captures some of the estimated variance originally interpreted as jumps in the JD model, it does not render the jumps irrelevant. Indeed, based on the decreased values of λ, and notable increases in θ, the average jump size, one conceivable theory is that the small perturbations are captured by the GARCH model while the larger discontinuous moves are represented as jumps.

4.2 Likelihood Ratio Test Results

Results of the pairwise likelihood ratio tests are presented in Table 3. Each entry in the table is a test of the form $LR_{X,Y}$, where the null hypothesis is that the appropriate stochastic process describing the data is X and the alternative hypothesis is that the appropriate stochastic process describing the data is Y. The probability that the null hypothesis is preferred to the alternative, based upon the Chi-squared distribution, is presented in the parentheses below each test statistics. The results indicate that the null hypothesis is rejected at the 1% level of significance in every case. Such a result is interpreted as demonstrating the statistically important gain in predictive power associated with adopting the added complexity associated with moving to specification Y, from X. Thus, for example, including GARCH improves upon the JD model, while folding in jumps improves upon the GARCH model. Importantly, the preferred specification allows from both jumps and time-varying volatility (GJD), for each of the three series.[15]

4.3 Implied Probabilities

Figure 3 gives a visual representation of the implied probabilities of at least one jump in the log price returns, occurring on each week, based on the JD model. The first panel shows the probabilities for the SREC solar credit returns. Given the frequency and persistence of implied probabilities that are well above 0.5, it is apparent that jumps play an important role in both SREC and electricity markets. We note too that

[15] One might be concerned that jumps in SREC prices are asymmetric, *i.e.*, that unanticipated positive shocks exert different effects than unanticipated negative shocks. To assess the potential for such effects on our results, we considered a variation on the GARCH(1,1) model that includes an asymmetric response parameter (known as "EGARCH"). We then collected the residuals from this approach and compared them against the residuals from the GARCH(1,1) process discussed in the main body of the paper, and found the two series were highly correlated. Then, we estimated a jump model using the residuals and compared the likelihood statistic from this estimation to the likelihood function from the EGARCH analysis, so as to evaluate the hypothesis that jumps are no longer statistically important when asymmetries are allowed. The test results strongly reject that hypothesis, corroborating the results outlined in the body of the paper. These results are available upon request.

the implied jumps close to 1 in the SREC markets are particularly common in 2012; this is intriguing as electricity prices in New Jersey were particularly large during this period. In contrast to the other two markets, the implied jump probabilities in the natural gas are smaller, and less frequently close to 1.

One possible explanation for this distinction is that the natural gas market is thicker than the other markets. In addition, the SREC market is noticeably less mature, having only come into fruition in the last fifteen years; by contrast, natural gas markets have been functioning for many years. Perhaps market thickness as well as maturity might create a form of insulation from large transitory events – thereby rendering jumps more important in younger or thinner markets, such as the SREC market. An alternative explanation is that gas can be stored while electricity can not, suggesting that both electricity prices and solar credits could be more prone to larger and more frequent jumps in price. In addition, natural gas supply tends to respond to gas demand; by contrast, in permit markets (such as the market for solar credits) the supply is commonly exogenous (and are fixed). The inability of supply to adjust to changes in demand in credit markets would then seem likely to induce greater variation in prices – along with a greater tendency for abrupt changes.

5 Solar Investment Under Uncertainty with Jumps

In light of the results above, which highlight the empirical importance of fat tails in solar credit price returns – and in particular the potential for jumps, a key issue here is the implication for investment in rooftop solar: do jumps influence investment decisions, and if so how? Addressing this issue requires an analysis of the investment decision with and without jumps; to that end, one needs to determine the value associated with forestalling the investment together with a determination of the value that is realized by investing. The former includes an "option value of waiting" to invest, an important element in the analysis. The latter is the expected present discounted value of the flow of rewards. Central to this investigation is a comparison of the value of forestalling the investment against the expected discounted flow just described. In this section, we provide such an analysis.

To illustrate the basic ideas, we adapt the analysis in Torani et al. (2016), who consider a problem where a decision-maker such as a homeowner is contemplating an investment in a solar project. The investment incurs an immediate cost that is irreversible, but that provides the decision-maker with an asset that generates a stream of benefits. In the solar installation case, the individual obtains a stream of solar credits, based on the volume of electricity the installed unit produces – these credits can be monetized in the solar credit market (with per unit value equal to the "solar renewable energy credit" we discussed above). The key features of this problem are that there is an economically significant up-front cost of investing, K, that is either irreversible or so expensive to reverse as to be qualitatively irreversible; and a stream of future rewards that depends on the per-unit value (or price) of the solar credit, which Torani et al. (2016) assume follows a GBM process.

5.1 Model formulation

The key stochastic ingredient in this framework is the price of a solar credit, which we denote by P. Following Torani et al. (2016), we assume the amount of electricity produced from the investment is a constant, which we denote as q; the flow of payments to the household associated with the solar credit is Pq . We start by working through the problem when P follows a GBM process – as in their model. Later, we analyze the problem when these processes are also subject to the potential for jumps.[16]

Assuming P follows a GBM process, its stochastic evolution is

$$dP/P = \mu dt + \sigma dz, \tag{15}$$

where dz is an increment of a Wiener process, μ is a deterministic drift term and σ characterizes the magnitude of stochastic variability in P. This form implies that changes in P are log-Normally distributed. It turns out that both the expected value and variance of P are increasing in both μ, and that the variance of is increasing in

[16] One aspect of the GBM process is that changes tend to exert an effect for a considerable length of time. An alternative approach would be to use a model in which the effect of changes in prices tend to dissipate relatively more rapidly – for example, a mean-reverting process. Analysis such a process is more complicated, though the broad principles we describe in this section still apply Dixit and Pindyck (1994).

σ.[17]

Upon execution, the value of an investment, $V_0(P)$, is the expected flow of future payments:

$$V_0(P) = \mathcal{E}\left\{\int Pqe^{-\rho t}dt\right\},$$

where ρ is the discount rate and $\mathcal{E}$ is the expectations operator. This value is readily computed as

$$V_0(P) = \frac{Pq}{\rho - \mu}. \tag{16}$$

At any moment where the decision to undertake the investment has yet to be made there are two possible decisions: either invest now or wait. The decision to invest now yields the payoff $V_0(P) - K$, where K is the sunk cost of investing. The decision to wait earns a flow payoff of 0 (since no action has been taken there are no flow benefits), while the option value of investing in the future, $F(P)$, is retained.

Delay will deliver an anticipated change in $F(P)$ (which can be thought of as the anticipated capital gains) less the foregone capitalized option value (which can be thought of as the interest earned on the net returns). If delaying is optimal, the fundamental equation of optimality requires that the optimal value function satisfies

[17] Expressing the initial value of P as P_0, the expected value of $P(t)$ can be expressed as:

$$\mathcal{E}[P(t)] = P_0 e^{\mu t},$$

while the variance can be shown to equal

$$\mathcal{V}[P(t)] = P_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1).$$

See Dixit and Pindyck (1994), pp. 71 – 72 for further discussion.

(Dixit and Pindyck, 1994):

$$\rho F(P) = \frac{1}{\mathrm{dt}} E\big[\mathrm{d}(F)\big] \tag{17}$$

where the expression on the right-hand side is the so-called Itô operator. The left-hand side of eq. (17) measures the capitalized option value, while the right-hand side is the sum of flow payoffs plus anticipated capital gains.

We proceed by expanding the expression on the right-hand side, which gives rise to[18]

$$\frac{1}{\mathrm{dt}} E\big[\mathrm{d}(F)\big] = \mu P F'(P) + \frac{1}{2}\sigma^2 P^2 F''(P). \tag{18}$$

Using eq. (18), we may express the fundamental equation of optimality as

$$\frac{\sigma^2}{2} P^2 F''(P) + \mu F'(P) - \rho F(P) = 0. \tag{19}$$

The solution to this equation is a power function:

$$F(P) = aP^b, \tag{20}$$

where b is the solution to the quadratic $\mathcal{Q}(b) = \big(\frac{1}{2}\sigma^2\big)b(b-1) + \mu b - \rho = 0$. Since $\mathcal{Q}$ is convex and both $\mathcal{Q}(0)$ and $\mathcal{Q}(1)$ are negative, there is one negative and one positive root (which exceeds one) to this quadratic; of these, the negative root is economically irrelevant and so the economically meaningful root exceeds one (Dixit

[18] One way to think about this step is that a Taylor's series expansion has been executed, with higher order terms dropped; see Dixit and Pindyck (1994) for details.

and Pindyck, 1994). It is straightforward to show that this root depends positively on σ and negatively on μ.

The value function $F(P)$ can be interpreted as the value of an option to invest in the future (Dixit and Pindyck, 1994). This option value is compared against the net benefits from acting (investing) at each time t (*i.e.*, V_0). It is optimal to invest when the net benefit from acting now just equals the option value of waiting; this implies a cutoff value P^*, which is implicitly defined by the "value-matching" condition:

$$F(P^*) = V_0(P^*) - K.$$

Together with the "smooth-pasting" condition

$$F'(P^*) = V_0'(P^*),$$

this determines the optimal time to invest:[19]

$$P^* = \frac{b(\rho - \mu)}{b - 1} K. \tag{21}$$

Since investment is delayed until P rises to this cutoff value, investment will tend to be undertaken sooner the larger is μ or the smaller is σ. These features can also be characterized in terms of the option value. Because a larger option value raises

[19] From eq. (16) $P^* V_0'(P^*) = V_0(P^*)$, while from eq. (20) $P^* F'(P^*) = bF(P^*)$. Using these facts, combining the value-matching and smooth-pasting equations then yields eq. (21).

the benefits from delay, it will tend to push back in time the moment at which the decision to invest is taken. Intuitively, an increase in the variance of the stochastic process raises the option value because of the potential for a more dramatic future increase in the underlying value P; delaying investment allows the decision maker to strategically take advantage of such future movements. This effect is more important the larger is the initial investment K.

Now suppose the value P evolves according to the mixed jump-diffusion process. Here, we assume changes in P are composed of two types of changes: 'typical' fluctuations, represented through the GBM process, and 'abnormal' fluctuations, due to the arrival of new information or some unusual event. We model the arrival of these abnormal fluctuations as following a Poisson process.[20] Letting n_t denote the number of such events that have occurred as of time t, the change in n_t during the interval $(t, t + \Delta t)$ is described by

$$dn_t = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ 1, & \text{with probability } \quad \lambda dt, \end{cases} \tag{22}$$

where $\lambda > 0$ is a parameter measuring the arrival frequency

We denote the size of a jump at time t, should one occur, as J_t; this is a random variable which for convenience we assume is independently and identically distributed

[20] Some authors model price jumps using a Lévy process, an approach that requires an *ex ante* definition of a jump. For example, Benth et al. (2008) define a jump as an observation that falls outside of 2 standard deviations from the mean. Other authors assume jumps follow a Poisson process; one advantage of this approach is that there is no need to arbitrarily define a jump *ex ante*.

as a lognormal random variable – *i.e.*, that $\ln(J)$ is Normally distributed. The resultant stochastic process for the random variable X may then be written as

$$dP/P = \mu dt + \sigma dz + J dn. \tag{23}$$

It can be shown that the expected value of the third term in eq. (23) is $\lambda\theta$, so that the drift term in the expressions for the evolution of $ln(P)$ becomes (Wilmot, 2010):

$$\tilde{\mu} = \mu + \lambda\theta. \tag{24}$$

In this setting, the equation governing the optimal value function is more complicated that we discussed above. Here, the equation governing the optimal value function is determined by the interaction between jump size, J, and continuation value, V:

$$\frac{1}{2}\sigma^2 P^2 F''(P) + \tilde{\alpha} P F'(P) + \lambda \int_0^\infty F(PJ)G(J)dJ = 0, \tag{25}$$

where $G(J)$ is the probability density function governing jump size. This expression can not generally be solved analytically; moreover, one can not apply the value-matching and smooth-pasting conditions (Dixit and Pindyck, 1994, p. 86).

An alternative explanation for the "fat tails" that are often observed for many energy commodities – in particular, the commodities we study in this paper – is that those prices are subject to time-varying volatility. An example of such a phenomenon is the "generalized autoregressive conditional heteroskedastic" (GARCH) framework.

Adapting the pure diffusion model to allow for this form of time-varying volatility gives a GARCH – diffusion process, under which the component σ in eq. (15) is replaced by a time-varying component h_t.[21] Again, this complication renders the problem sufficiently complicated as to preclude obtaining an analytic solution. Accordingly, we resort to numerical methods in the discussion below, and so we proceed using a numerical simulation.

5.2 Simulation Results

To facilitate numerical simulations, we must first specify the discount rate ρ; the mean α and standard deviation σ of the GBM formulation; and the jump intensity λ associated with the Poisson process. In our baseline simulations, we set these parameters as $\rho = 0.02, \alpha = 0.04, \sigma = 0.2$, and $\lambda = 0.10$. The distribution governing Y, the magnitude of a jump (should it occur), is assumed to be lognormal – *i.e.*, $\ln(Y)$ is Normally distributed – with mean $\theta = 0$ and standard deviation $\delta = 1$.

For a given parameterization, we solve for the critical value associated with in-

[21] The process in question is:

$$h_t \equiv E_{t-1}\left(\sigma^2\right) = \kappa + \alpha_1 \left(x_{t-1} - \mu\right)^2 + \beta_1 h_{t-1}.$$

In this formulation, the variance at a point in time is a combination of a time-invariant term (κ) and a weighted combination of the sample variance from last period (the second term, with weight α_1) and the variance from the preceding period (the third term, with weight β_1). One advantage of using such a formulation is that it allows for the underlying variance to change over time. This form is characterized by four parameters, μ, κ, α_1 and β_1. There is a general consensus in the literature is that a GARCH model with a limited number of terms performs reasonably well, and so we restrict our focus to this more parsimonious representation.

vesting; the interpretation is that when the expected value from investing meets or exceeds this critical value, the investment will be taken. This critical value will correspond to the sum of the investment cost itself and the option value of waiting. The difference between the critical value and the requisite up-front investment may then be interpreted as the option value of waiting. We also calculate the ratio of the critical value to up-front investment cost.

Our first set of simulations investigates the role played by the jump intensity. Here, we vary λ between 0 and 0.2, by increments of 0.05; results from this set of simulations are summarized in Figure 4. This figure displays the option value associated with delaying investment, for various levels of up-front investment (*i.e.*, K) across the possible values of λ. The first feature we observe is that the option value of delaying investment rises as the amount of money that must be invested increases. This is intuitive: because larger investments require risking more money, the decision-maker is more cautious about undertaking the investment. In these simulations, the tendency to delay investment tends to be more pronounced as the probability of a jump increases: while largely insensitive to λ when the required investment is small, the option value of waiting does respond to increased jump intensity at larger investment levels.[22] Moreover, we note that the impact of increasing λ is most pronounced at small value of λ. In particular, the largest effect appears to occur when the probability of a jump occurring is increased from 0 to a positive value – *i.e.*,

[22] One should not make too much of the seeming equivalence of option values at the smallest level of K: The numerical grid we employ in the solution algorithm is not sufficiently granular to detect differences between option values at small levels of K.

when one allows for the possibility of jumps. Indeed, this effect become ever-more important as the up-front cost is increased.

In these simulations, an increase in λ raises the variance of the stochastic process. As we noted above, an increase in the variance of the stochastic process tends to increase the option value of waiting; delaying investment allows the decision maker to capitalize on such future movements.

In the second set of simulations, we vary the expected value of the natural log of jump size (θ), allowing for values ranging from -0.2 to 0.2 (in increments of 0.1). In this way we consider cases where abrupt movements in prices are negative on average as well as cases where jumps are positive on average. The results from this simulation are presented in Figure 5. As in the first set of simulations, we note that option value of delaying the investment rises as the amount of money that must be invested increases. Referring back to eq. (24), an increase in θ will raise the drift in the stochastic process. This induces conflicting effects on the option value of waiting: on the one hand, larger drift depresses the option value of waiting; on the other, larger values of θ raise the variance of X, and larger variance increases the option value of waiting. In the simulations we report here the former effect appears to be somewhat larger, though the net effect is small. Evidently, the average jump value exerts a less significant influence on the value of delaying investment than does the potential for a jump in the first instance.

The third set of simulations we consider varies the standard deviation of the jump size (δ); here we consider values ranging from 0.5 to 1.5, in increments of 0.25. Results from these simulations are presented in Figure 6. As noted above, raising the variance of the jump size pushes up the variance of the stochastic variable X, which induces an increase in the option value of waiting. Interestingly, while this effect is small when $\delta < 1$, it becomes more pronounced when $\delta > 1$. That is, while the option value of waiting is not particularly responsive to changes in the variance of jump size at smaller levels of that variance, the impact upon option value becomes significantly more pronounced when the variance of the jump size increases above unity. Indeed, variations in the potential size of the jump play an ever-larger role as the amount of money that must be invested increases. Again, this seems intuitive: when prices are subject to possible jumps with particularly large variation, the impact on the value of waiting increases to an ever-larger degree – generating an increasing motive to delay. That is, greater variation in jump sizes make waiting more attractive, and hence raise the option value at the optimal investment time.

6 Discussion

In this study we investigated the price of solar renewable energy certificates (SRECs) in New Jersey – production credits awarded to owners of grid-connected solar PV systems – allowing for the potential presence of jumps and time-varying volatility. Our results point to the importance of both of these features in modeling SREC price returns. In addition to providing a statistically important improvement in explanatory power of the empirical model, our results allow us to simulate the implied probability that at least one jump would occur in any given month. Following this approach, we developed implied jump probabilities for price returns for SRECs, as well as natural gas and electricity price returns. Our results point to the consistently important role of jumps in both SRECs and electricity prices, but less so for natural gas prices. Jumps in SRECs appear to have been of particular significance between late 2011 and early 2013, a period when electricity prices in New Jersey were particularly high – presumably providing heightened incentives for homeowners to invest in rooftop solar. Two important factors that distinguish SREC and natural gas markets are the relative thickness of the market and its maturity. Natural gas markets are commonly thought to be quite thick and mature, whereas SREC markets have been in place for a much shorter period, and there are far fewer suppliers of solar projects. This points to the potentially important role of market structure in driving fat tails in price returns.[23]

The rapid growth in solar PV capacity has been driven by the falling price of installing PV and the implementation of public policies (Nemet, 2019). Even so, rooftop solar panel installations are a non-trivial expense to the homeowner; Dastrup et al. (2012) estimate the cost to be on the order of 3-4 % of the home value. Perhaps because of this feature, Mamkhezri et al. (2020, p. 178) have argued that "there is a

[23] Of course, market structure can exert an important influence on price levels – either by allowing sellers to charge higher prices or by allowing sellers to capitalize on cost reductions, for example through scale economies (Gillingham et al., 2016). Our point is that more concentrated markets may be more susceptible to the influence of unexpected events that could cause dramatic changes in prices.

diminishing return in support for [r]ooftop solar." As we noted above, the presence of uncertainty can retard investment, particularly in projects with largely irreversible upfront investments such as rooftop solar installations. The obvious alternative would be a large-scale solar farm; but the private incentive to undertake such a large investment also depends importantly on the option value of waiting to invest. This delaying effect is larger the larger is the variation in the stochastic element of concern (here, SREC prices). Importantly, the presence of jumps mimics an increase in variability. As such, our finding that both jumps and time-varying price volatility provide a statistically important explanation of the pattern in SREC prices is of some concern, as there is reason to fear these effects will increase the option value of waiting – and thereby retard the rate of investment in solar energy. The upshot is that development of solar farms and similar investments are likely to occur at a somewhat slower pace than one might prefer – slowing the energy transition.

This effect is reminiscent of the potential delay in investment that arises when learning-by-doing effects are important (van Benthem et al., 2008). When learning-by-doing effects are present, private decision-makers undervalue the potential future benefits associated with present actions, *i.e.* they overweight current effects. In contrast, when investments the option value of waiting induces delayed investment, decision-makers see increased present gains in part because of the desire to capture potential future gains that might arise if jumps occur.

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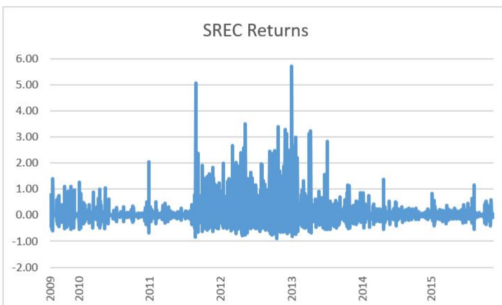
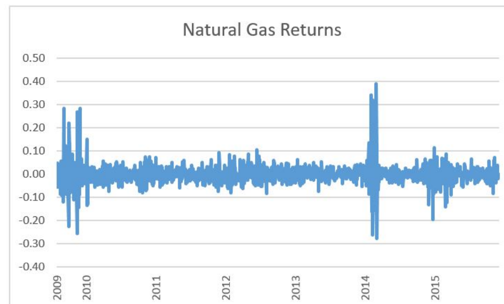
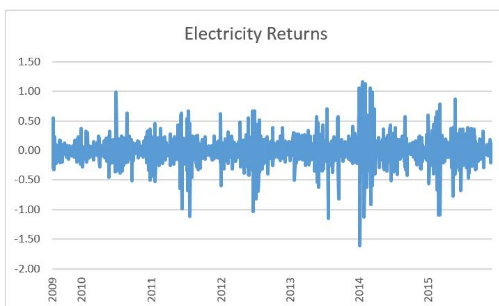
Figure 1: Time series of price returns



(a) SREC



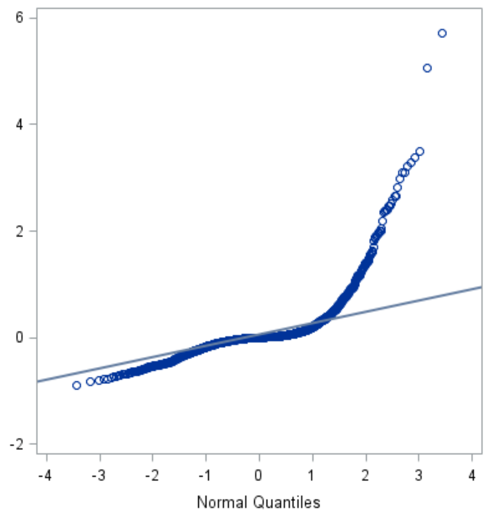
(b) Natural gas



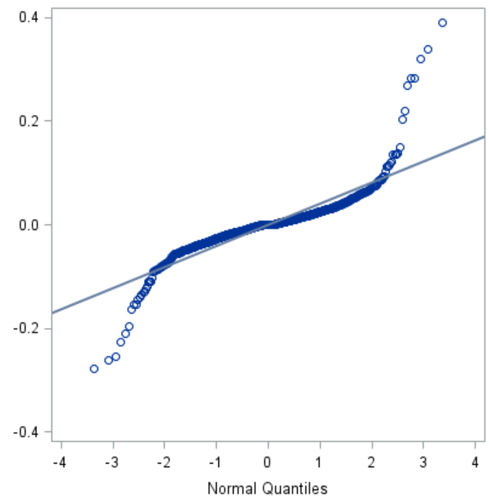
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

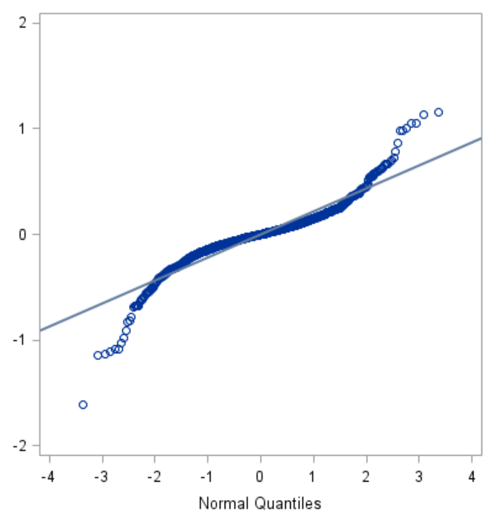
Figure 2: Quantile–quantile plots



(a) SREC



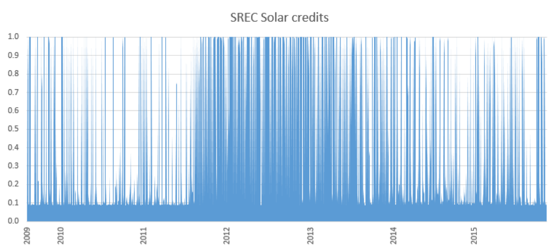
(b) Natural gas



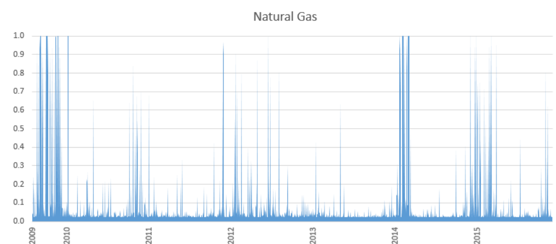
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

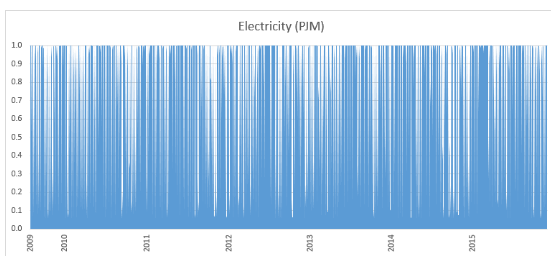
Figure 3: Fitted probabilities based on the Jump Diffusion model results



(a) SREC



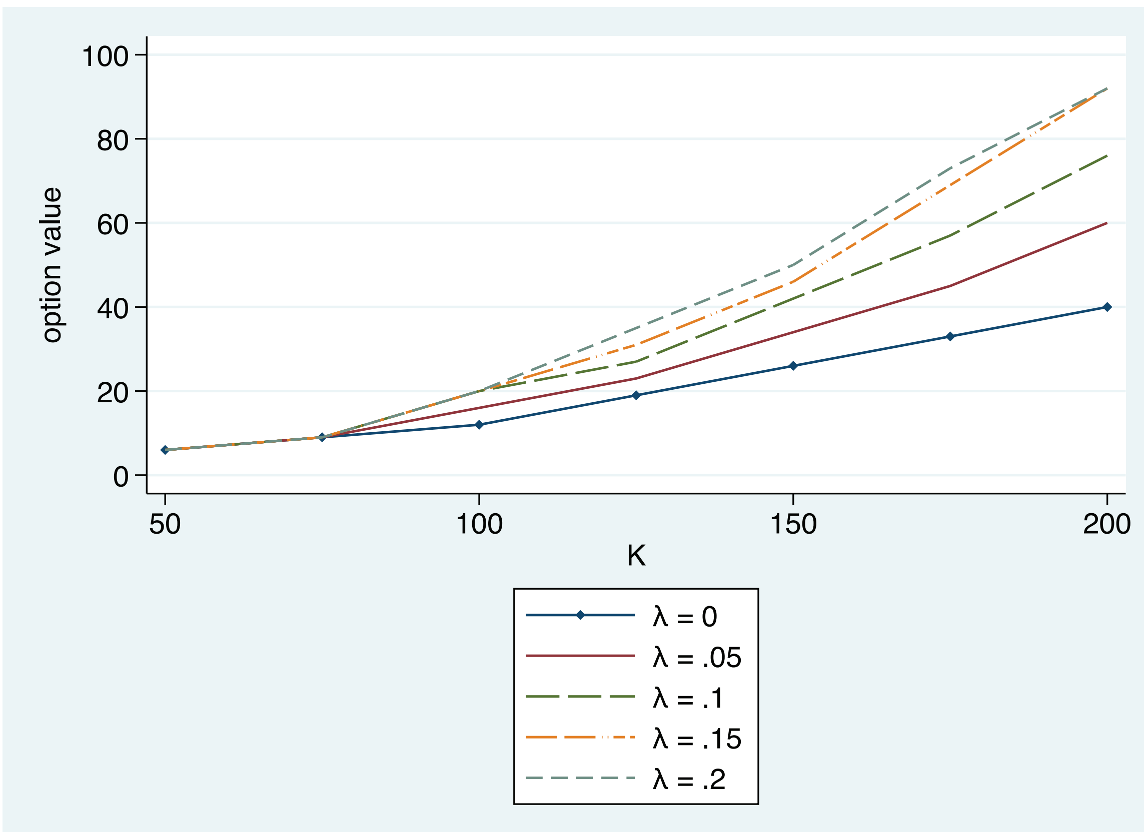
(b) Natural gas



(c) Electricity

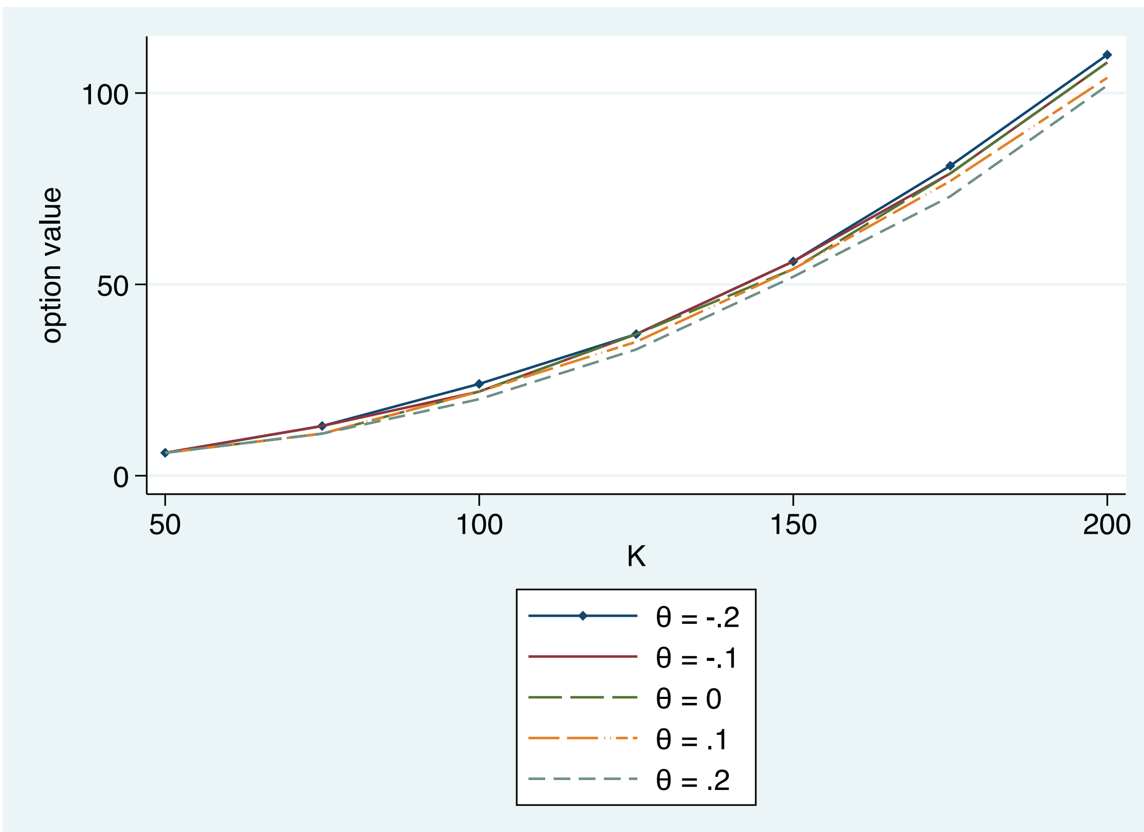
Source: Authors' calculations, based on results from Table 2.

Figure 4: The impact of jump probability upon the option value of waiting.



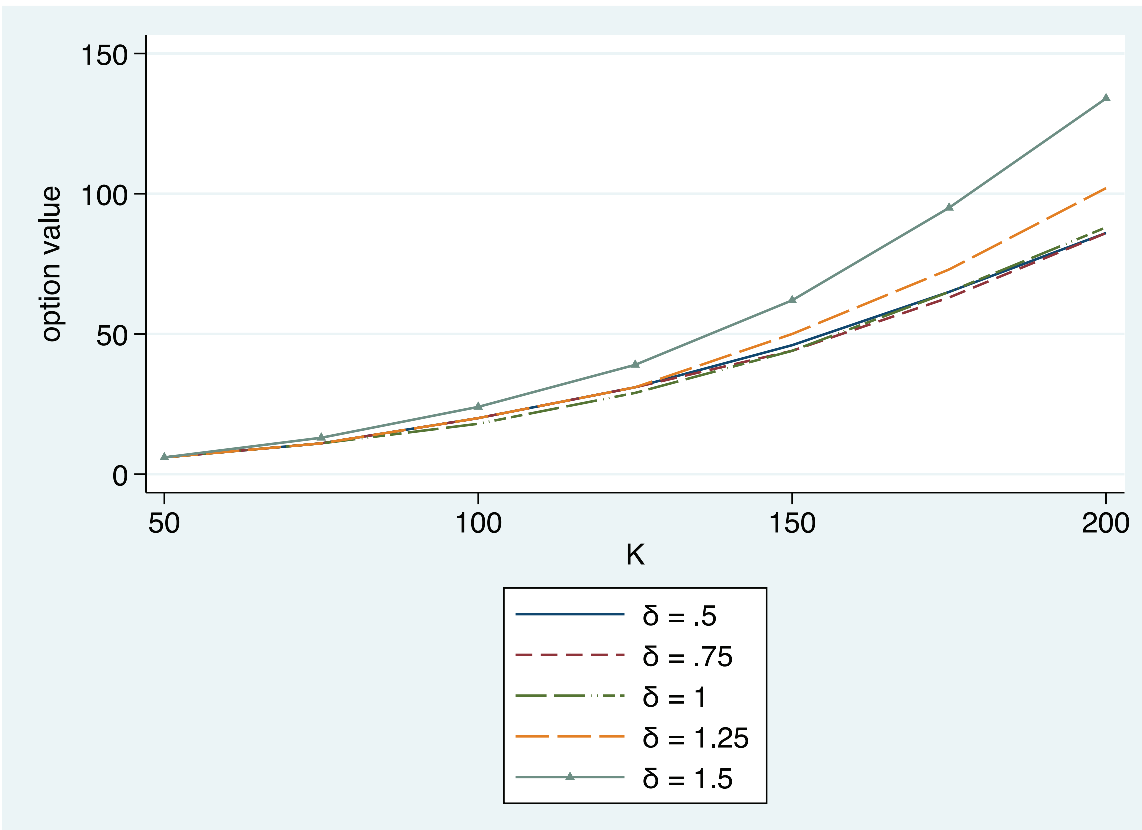
Source: Authors' calculations.

Figure 5: The impact of jump mean upon the option value of waiting.



Source: Authors' calculations.

Figure 6: The impact of jump variance upon the option value of waiting.



Source: Authors' calculations.

Table 1: Summary Statistics: Solar Price Returns

Variable	Solar Returns	Natural Gas Returns	Electricity Returns
Sample Range			
Start	Aug 01, 2009	Aug 01, 2009	Aug 01, 2009
End	Nov 30, 2015	Nov 30, 2015	Nov 30, 2015
Summary Statistics			
Mean	0.0648	-0.00029	-0.0003
Median	-0.0014	0.00000	0.00000
Minimum	-0.8867	-0.27844	-1.6136
Maximum	5.7102	0.39007	1.1624
Variance	0.2137	0.00166	0.0471
Std. Dev.	0.4623	0.04071	0.2171
Coeff. of Variation	713.6	-14,172.5	-84,591.0
Skewness	4.1873	1.11170	-0.3023
Kurtosis	29.4956	20.22889	7.6107
n	2099	1599	1606
Test of Normality			
Kolmogoro Smirnov	0.2508	0.1133	0.1017
p-value	<0.01	<0.01	<0.01
Unit Root Test			
Modified Dickey Fuller	-4.806	-4.909	-4.446
1% critical-value†	-3.48	-3.48	-3.48
Lags	20	23	24

Note: Statistics are based on a sample of daily observations. Solar returns are based on SREC prices; natural gas returns based on Henry Hub spot prices; and electricity returns are based on PJM prices.
†: Critical values are based on Elliot et al. (1996).

Table 2: Estimation of the Parameters for Solar Price Returns

Variable	μ	σ	κ	α_1	β_1	λ	θ	δ
				Solar Credits				
PD	0.0648***	0.4622***	.	.	.	.	.	.
	(0.010)	(0.007)						
JD	-0.0069***	0.0536***	.	.	.	0.6673***	0.1074***	0.4540***
	(0.002)	(0.032)				(0.037)	(0.014)	(0.17)
GPD	0.0236***	.	0.0011***	0.1084***	0.8816***	.	.	.
	(0.004)		(0.000)	(0.009)	(0.009)			
GJD	0.0017	.	0.0007***	0.3899***	0.3216***	0.2065***	0.3818***	0.6744***
	(0.001)		(0.0001)	(0.058)	(0.050)	(0.019)	(0.056)	(0.045)
				Natural Gas				
PD	-0.02872	4.0695***	.	.	.	.	.	.
	(0.043)	(0.072)						
JD	-0.0529	2.3915***	.	.	.	0.0988***	0.2385***	10.3349***
	(0.046)	(0.076)				(0.019)	(0.416)	(9.63)
GPD	-0.0829	.	0.2716***	0.1404***	0.8431***	.	.	.
	(0.063)		(0.068)	(0.017)	(0.017)			
GJD	-0.0704	.	0.5013***	0.1160***	0.8010***	0.0195***	0.9577 ***	10.3349***
	(0.062)		(0.105)	(0.016)	(0.024)	(0.007)	(2.77)	(3.029)
				Electricity Prices				
PD	-0.0449	21.7033***	.	.	.	.	.	.
	(0.090)	(0.383)						
JD	0.2161	10.8724***	.	.	.	0.3169***	-0.7611	32.7599***
	(0.725)	(0.591)				(0.055)	(1.358)	(2.822)
GPD	0.3107	.	19.6518***	0.2463***	0.7350***	.	.	.
	(0.332)		(3.879)	(0.029)	(0.026)			
GJD	-0.8772	.	7.9596**	0.2219***	0.6945***	0.1654***	10.3549***	21.7218***
	(0.490)		(5.520)	(0.027)	(0.032)	(0.062)	(2.836)	(4.758)

The standard errors are presented in the parentheses.
***, **, and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Table 3: Likelihood Ratio Test Results

Variable	$LR_{PD,JD}$	$LR_{PD,GPD}$	$LR_{JD,GJD}$	$LR_{GPD,GJD}$
Solar Credits ($SREC$)	2539.02	1714.60	570.62	1395.04
	(0.000)	(0.000)	(0.000)	(0.000)
Natural Gas (HH)	812.45	999.34	254.05	67.16
	(0.000)	(0.000)	(0.000)	(0.000)
Electricity (PJM)	540.64	634.21	246.59	153.02
	(0.000)	(0.000)	(0.000)	(0.000)

Note: Based on a sample of 2009 observations. *p-values* are given in the parentheses.