THE TRANSITION TO RENEWABLE ENERGY

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Abstract. The existing economics literature neglects the important role of capacity in the production of renewable energy. To fill this gap, we construct a model in which renewable energy production is tied to renewable energy capacity, which then becomes a form of capital. This capacity capital can be increased through investment, which therefore comes at the cost of reduced consumption. We describe how society could optimally elect to split production between immediate consumption and investment in the two forms of capital. Our model delivers an empirically satisfactory explanation for simultaneous use of non-renewable and renewable energy. While transitioning to renewable energy can often ensure perennially increasing utility, there are combinations of initial levels of the non-renewable resource and renewable capacity that can generate Malthusian-like effects, with utility declining at some point in time.

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1. Introduction

The tension between mounting pollution and the increasing need for energy that has occurred over the past several decades raises important questions about the potential well-being of future generations. These questions are squarely in line with the Malthusian tradition, as they lead us to ask whether future societies are doomed to accept a lower standard of living. One possible pathway out of this gloomy forecast is tied to the rise of renewable energy. Might it be the case that transitioning from dirty, fossil-fuel based energy to clean, renewable energy could allow continuing growth while avoiding the damages that have been historically associated with energy use?

This question is directly linked to the concept of sustainability (Pezzey and Toman, 2002), which economists have debated for many years. Much of this literature interprets sustainability as non-decreasing well-being of a typical member of society (Solow, 1991). To operationalize this concept, the literature typically employs economic growth models. A general finding is that for future generations to be at least as well off as current generations, society must invest the rent from non-renewable resource use to increase the stock of physical capital (Hartwick, 1977). Relatedly, when consumption of a non-renewable resource is associated with pollution, as with fossil fuels, society is motivated to transition to an alternative, more sustainable, resource.[1]

In general, economists have modeled this sort of transition by contrasting resource use from a non-renewable source against the use of a "backstop" technology. The backstop is usually assumed to be able to deliver any amount of energy at a constant marginal cost, which implies the resource use can be expanded to the extent society desires without increasing its marginal cost. Models with constant marginal cost of renewable energy commonly yields a

[1] See, for example, Withagen (1994). There is a conceptual link between sustainability and the use of a non-renewable resource whose usage generates pollution (Jevons, 1865; Hartwick, 1977; Forster, 1980; Xepapadeas, 2005).

result whereby society transitions between non-renewable and renewable resources when the marginal cost of the former (including any scarcity rents or pollution charges) has risen to equal equals the marginal cost of the latter (Hoel and Kverndokk, 1996). This switch typically occurs in a once-and-fo-all fashion, which is plainly at odds with reality. Furthermore, for many renewable resources, the associated marginal cost of production is zero (or close to it); in addition, most of the costs associated with renewable energies are sunk: it is expensive to build the capacity to generate energy from renewable resources (Energy Information Administration, n.d.).[2] This generating capacity then constrains the amount of renewable energy that is available; to increase renewable resource use, capacity must be expanded – which, as figure 1 illustrates, has happened over the past several years.

In addition, a common feature of the backstop resource model – and indeed many resource extraction models – is that society fully depletes the stock of the nonrenewable resource, before switching to renewable resource use.[3] As such, there is no potential for simultaneous use. But many contemporary economies simultaneously utilize both non-renewable and renewable energy.

In this paper, we address these empirical inconsistencies by presenting a model that more satisfactorily characterizes the role of renewable energy. We adapt a neo-classical growth model by incorporating an energy input into the aggregate production function; the energy input can be associated with a (dirty) non-renewable resource, such as coal or oil, or a (clean) renewable resource, such as wind or solar.[4] Our innovation is to impose a constraint on the rate of usage of the renewable energy input, which we interpret as the

[2] The US Energy Information Administration (Energy Information Administration, n.d.) lists the levelized cost of electricity (LCOE) for new energy sources to come online in 2020. The LCOE breaks down into capital costs, fixed and variable operations and maintenance (O&M) costs and transmission costs. For wind and solar energies, variable O&M costs are zero while for hydroelectricity, they only account for 8.4% of the total LCOE. When sunk initial capital costs and fixed O&M costs are combined, they represent from 89% to 98% of the LCOE for renewables (wind, solar and hydro electricity). This compares to less than 35% for the multiple technologies of gas-fired plants, and up to about 74% for newest coal-fired plants.

[3] Switching too early would imply forgoing low-cost energy, which helps boost the economy and increases consumption.

[4] In this regard, our paper is directly related to the literature on green growth. For a discussion of the green growth literature see Smulders et al. (2015).

renewable capacity. Similar to man-made capital, this capacity can only be increased through investment (National Renewable Energy Laboratory, 2004; Knapp and Jester, 2001). We allow the efficacy of this investment to depend on the stock level, reflecting learning-by-doing. Working against any increases in capacity is wear and tear from the use of the renewable energy, which is tied to the rate of usage of renewable energy (Staffel and Green, 2014; Jordan and Kurtz, 2013). In this way, our model provides a more realistic description of the role played by renewable energy in the time path of society's well-being – and hence the implications for sustainability – than can be found in the extant literature. Naturally, the presence of capacity constraints implies the need to introduce a state variable that measures capacity; this feature is absent from all the papers discussed above. A key feature of our paper is the incorporation of this form of capital.

When the generation of energy from renewable sources is constrained by the installed capacity at any given time, it is entirely possible that there will be a period when both types of resource are used simultaneously. Expanding the renewable capacity would ease this constraint, leading to a period in which the share of renewable resource use rises over time, as society transitions away from the non-renewable resource base. Such a pattern is fully in line with reality: for example, for nearly 15 years the role of coal in producing electricity in the United States has been steadily shrinking, while the share attributable to wind and solar energy has been rising.

Others have addressed the issue of simultaneous use. When pollution is linked to the non-renewable energy use, the associated social costs are accounted for; this can lead to a phase where both types of energy resource are used simultaneously, even if non-renewable energy and the renewable alternative are perfectly substitutable (Tahvonen, 1997; Jouvet and Schumacher, 2012). This approach implicitly assumes that society assesses some form of charge against the production of dirty energy; while one can find isolated examples of such charges, a number of important economies do not penalize dirty energy use. Simultaneous use can also occur when the marginal cost of the renewable backstop is increasing. When both the energy demand and the cost of renewable energies are low, society may use renewables only first, then switch to simultaneous resource use before switching again to renewables in either finite or infinite time. Some non-renewable energy stock may be left in the ground, implying a phase in which the economy only uses renewable energies (Tahvonen and Salo, 2001). But as noted above, the marginal cost of renewables are commonly small – and non-increasing – over wide ranges of output levels.

van der Ploeg and Withagen (2014) link the initial levels of capital, pollution and non-renewable resource stock to the type of energy used and the order in which they are used. They find that simultaneous energy usage always follows a phase where only the non-renewable resource is used; this only happens after man-made capital is above its carbon-free steady-state level. The non-renewable resource is never phased out and man-made capital has to be reduced to reach its long-run level – in this sense the economy "overshoots." According to this view, simultaneous use occurs as the man-made capital stock is drawn down. This feature of their model is arguably at odds with the current empirical reality, as society is currently using both renewable and non-renewable energy at the same time that man-made capital seems to be expanding.

Picking up on observations of simultaneous energy use, Gronwald et al. (2017) study energy production when the renewable backstop to dirty energies is constrained. Similar to our model, they assume that society cannot completely switch to renewable energy as it is constrained by capacity. In their model, however, renewable capacity is very hard to adjust; when they do consider an expansion in capacity it is treated as an exogenous one-time shift. That aspect of their model is inconsistent with the empirical evidence in figure 1, which reveals a gradual (and presumably endogenous) expansion of renewable capacity. By contrast, our model does allow for such an endogenous expansion in renewable capacity. In line with the economists criticisms of Malthus' gloomy predictions, van der Meijden and Smulders (2018) study technological progress and R&D in the transition to renewable energies. They find that subsidies on green technologies work better than a carbon tax to expedite the transition to renewable energies. The desire for a smooth transition yields a simultaneous use of both energy resources. In our model, this is not so much a desire, but a necessity. Without prior investment in renewable energy capacity, society cannot ensure a 'sustainable' transition.

While not incorporated in our analysis, an important issue with renewable energy is that it is intermittent, which impacts its ability to act as perfect substitute to non-renewable energy. If storage is possible, renewable production during off-peak times can cover peak daytime demand (Pommeret and Schubert, 2018). This would allow for a smooth phase-out of non-renewable energies, with solar and wind energies stored for off-peak consumption. With high intermittency issues, complete transition to renewables while both non-renewable and renewable energies act as a buffer energy source (Sinn, 2017).

The paper is organized as follows. We start in section 2 by discussing the main features of our model. In section 3, we characterize the optimal paths of consumption and renewable capital in the basic model. We discuss a numerical simulation of the model in section 5. A discussion of the decentralized problem is presented in section 6. We discuss an extension of the model to include multiple capital stocks in section 4. Section 8 offers some concluding thoughts.

2. Modeling Preliminaries

Our model is built up from a conventional neo-classical growth model, but with some additional features. Societal well-being is based on aggregate consumption, C. To facilitate consumption, society must first generate an amount of aggregate production, Y. We investigate two variants of the problem. In the first, production is determined by an energy input E, with $Y = F(E)$. The energy input is comprised of the rate of non-renewable energy use q, which one can think of as oil, and renewable energy use h, which one can think of as solar:

$$E = q + h.$$

In the second, production is linked to two types of inputs: physical capital K and energy, and so $Y = F(K, E)$. Energy can be produced using renewable and non-renewable resources. We denote the rate of use of non-renewable energy input as q, and the rate of use of renewable energy input as h. For expositional clarity, we will often refer to the non-renewable energy input as "oil" and the renewable energy input as "solar" or "wind."

The rate of renewable energy use is based on the stock of renewable capital, X, which we interpret as generating capacity. The key wrinkle is that renewable energy is capacity-constrained: $h \leq X$.[5] Renewable energy has very low variable costs, which we set equal to 0. The use of non-renewable energy entails costs, which we model as flow fixed costs.

The problem we are addressing is potentially quite complex, and so to facilitate a deeper investigation we impose a number of assumptions on functional forms. None of these are controversial, but using them greatly simplifies that analytics.

First, to focus the discussion on the role of renewable energy we specify the production function as a power function. In the first variant, the production function is

A1 $F(E) = \alpha E^{\beta}$.

In the second variant, we take the production function to be Cobb-Douglas:

A1′ $F(E, K) = \alpha E^{\beta} K^{(1-\beta)}$.

Second, we assume that utility U satisfies standard neo-classical assumptions; again, to focus our attention we specify utility as iso-elastic:

A2 $U(C) = \frac{C^{(1-\theta)}}{1-\theta}$.

[5] One might also imagine that renewable energy can only come on line after the capacity has reached some threshold level, for example because of network effects. To the extent this aspect applies, it would likely induce a "pre period," during which only dirty energy was used – and where investment into the renewable capacity was required to build up the stock. As we observe significant levels of renewable energy being used currently, we focus our analysis on the period after that threshold stock has been reached. One way to envision invoking this implicit assumption is by requiring that the initial level of the renewable stock, $X(0)$, is large enough so as to surpass the threshold level articulated above.

Third, we assume that the rate of change in renewable capital depends on both investment R and the standing stock X. This latter feature could reflect learning-by-doing in the evolution and cost of technologies. Working in the opposite direction, there could be Ricardian effects at play if the ability of renewable capital to generate energy depends on the suitability of the land used for the capital installation.[6] We capture the role of X via a function $G(X)$, which we interpret as reflecting the effect of one dollar of investment, *i.e.*, $G(X)$ describes the efficacy of investing in renewable capital. This function plays an important role in our inquiry. We assume G is the function

$$G(X) = X^a e^{-bX}, \qquad \text{with} \;\; a > 0, b > 0; \tag{1}$$

so that gross additions to renewable capital are $RG(X)$. This functional form allows for learning by doing effects for low stock of capacity, while other industry-wide constraints tend to lower the marginal impact of investment in capacity for higher stock levels (Mundlak, 1964). Lastly, we assume the stock depreciates at a a constant rate δ_0.[7] Altogether, then, the rate of change in renewable generating capacity is given by:

A3 $\dot{X} = RG(X) - \delta_0 X = RX^a e^{-bX} - \delta_0 X$. In the pursuant discussion we occasionally refer to the elasticity of G with respect to X:

$$\epsilon_G = \frac{XG'(X)}{G(X)} = a - bX.$$

[6] Gerlagh (2010) assumes decreasing returns to scale in the provision of the backstop, on the grounds that the best placement sites are depleted – although he models this effect via increasing marginal cost of the clean backstop. Chakravorty et al. (2012) allow for learning-by-doing, which they capture by assuming that the unit cost of renewables decreases with accumulated backstop use.

[7] This depreciation can be substantial. For example, Staffel and Green (2014) find that for wind farms in the United Kingdom, load factors – the percentage of electricity actually produced, compared to the theoretical maximum – falls by roughly 1.5% per year. While less dramatic, depreciation rates for solar photovoltaics are on the order of 0.5% per year (Jordan and Kurtz, 2013).

Fourth, we assume the pollution stock induces "damages" $D(P)$, with positive marginal damages.[8] We assume these damages are quadratic in the pollution stock:

$$D(P) = \frac{d}{2}P^2. \tag{2}$$

For analytic expediency, we follow van der Ploeg and Withagen (2014) by assuming there is no depreciation in P. Accordingly, since cumulative extraction at any point in time t is $S_0 - S(t)$, the pollution stock at any point in time can be written as

$$P(t) = P_0 + \kappa\big(S_0 - S(t)\big).$$

Pollution damages are then

A4 $D(P) = \frac{d\kappa^2}{2}(\frac{P_0}{\kappa} + S_0 - S)^2$.

Fifth, we assume the stock of remaining non-renewable resource at time t, $S(t)$, is depleted with the rate of usage of non-renewable energy; so that we might measure the stock in terms of natural units for damages (*e.g.*, volume of CO_2) while measuring the flow in terms of energy (*e.g.*, gigajoiules), we allow for a constant that captures the conversion between units of measurement:

$$\dot{S} = -q. \tag{3}$$

We assume oil extraction entails a constant cost (*i.e.*, a flow fixed cost) f as well as a constant marginal cost γ, so that extraction costs are

$$c(q) = \begin{cases} f + \gamma q & \text{if oil is used } (q > 0) \\ 0 & \text{if oil is not used } (q = 0). \end{cases}$$

We envision a central decision-maker, or "social planner," who is charged with promoting the aggregate well-being of society for all time – the present discounted flow of aggregate

[8] A natural interpretation here would be the contribution to atmospheric carbon stocks associated with burning fossil fuels. As a second example, oil sands extraction in Alberta, Canada leaves significant tailings, in particular containment ponds retaining water with fine or toxic residues (Heyes et al., 2018).

utility, less any costs from extracting oil, less any damages from the pollution stock. The discount rate is ρ.[9]

3. Problem where oil production is proportional to stock

We start our discussion with an analysis of a simple version of the problem, where oil production is proportional to the remaining stock of oil. In this discussion we summarize the key elements of the solution, and relegate most of the analytic details to Section A.1 of the Appendix. One can think of this formulation of production as a variation on the notion of a "decline curve."[10] With this assumption, the evolution of the stock of remaining oil is

$$\dot{S} = -q = -\lambda S.$$

where we denote derivatives with respect to time by a dot over the associated variable. Note that the solution to this differential equation is

$$S(t) = S_0 e^{-\lambda t}, \tag{4}$$

which then implies oil production is

$$q(t) = \lambda S(t) = \lambda S_0 e^{-\lambda t}. \tag{5}$$

Then, since the oil stock decreases exponentially in this version of the problem during phase 1, we can write damages as

$$D(P(t)) = \frac{d\big(P_0 + \kappa S_0 (1 - \lambda e^{-\lambda t})\big)^2}{2}. \tag{6}$$

[9] This discount rate was the source of considerable controversy in the context of climate policy. For example, Stern (2007) proposes a very low discount rate, barely positive, while Nordhaus (2007) suggests a rate closer to 3%. The ultimate choice of discount rate implies an ethical judgment, which is beyond the scope of our paper and discussion. We merely point out the nature of the dynamic optimization scheme our mythical social planner must decide upon, and relate it to a notion of sustainability.

[10] Anderson et al. (2018) and Mason and Roberts (2018) provide empirical evidence supporting this assumption. We discuss the extension to this basic model wherein q is endogenously selected in the next section.

The planner's problem in this context is to select time paths of consumption level, renewable energy use and investment in renewable capital so as to maximize the discounted flow of net benefits over an infinite horizon. The decision problem is constrained by the evolution of the renewable capital stock and the capacity constraint on renewable usage. The problem is also influenced by the accounting identity governing output. Output can be invested in renewable energy or consumed:

$$F(E) = C + R, \tag{7}$$

which allows us to write the equation of motion for X as

$$\dot{X} = G(X)\big(F(E) - C\big) - \delta_0 X.$$

In the pursuant discussion we will refer to the phase where oil is used as "phase 1" and the phase where oil is not used as "phase 2."

As we discuss below, the capacity constraint on renewables binds on the optimal path, so that $E(t) = X(t) + \lambda S_0 e^{-\lambda t}$ (resp., X) in phase 1 (resp., 2). Then from **A1**, we have:

$$F(E(t)) = \begin{cases} \alpha\big(X(t) + \lambda S_0 e^{-\lambda t}\big)^{\beta} & \text{in phase 1} \\ \alpha X(t)^{\beta} & \text{in phase 2.} \end{cases} \tag{8}$$

Extraction costs during phase 1 in this version of the problem are

$$c(q(t)) = f + \gamma\lambda S_0 e^{-\lambda t}, \tag{9}$$

The planner's objective is to maximize the discounted flow of utility, less the discounted flow of extraction costs and pollution damages:

$$\int_0^{\infty} \Big[U(C) - c(q) - d\big(P_0 + S_0 - S(t)\big)\Big] e^{-\rho t} dt$$

by choice of the time paths of consumption and investment in renewable capacity, along with the time of transition from phase 1 to phase 2 transition. Letting $\hat{T}$ denote the transition time, the planner's objective can then be written as:[11]

$$\max_{C,h,\hat{T}} \left\{ \int_0^{\infty} U(C)e^{-\rho t}dt - \int_0^{\hat{T}} D\big(P(t)\big)e^{-\rho t}dt - \Delta_1(\hat{T}) - \Delta_2(\hat{T}) \right\}, \tag{10}$$

subject to assumptions **A2**, **A3 and A4**, the exogenously determined time paths of q, S and pollution damages D, and the constraints $0 \leq h \leq X$ and $0 \leq C \leq F(E)$.[12] The terms Δ_1 and Δ_2 represent, respectively, the discounted flow of extraction costs during phase 1 and the discounted flow of pollution damages during phase 2, each of which is exogenously determined (aside from the chosen transition time $\hat{T}$) in the context of this variant of our model.

The optimal rate of consumption is characterized by

$$U'(C) = \phi_X G(X), \tag{11}$$

where ϕ_X is the shadow value of the renewable resource capacity. Eq. (11) states that optimal consumption equalizes the marginal utility from consumption to its current marginal cost is common.[13] The optimal rate of renewable energy use is dictated by complementary-slackness conditions; we show in Appendix A that these conditions imply the capacity constraint will bind.

The solution to this dynamic optimization problem also includes an equation of motion governing the shadow value ϕ_X, which is:

$$\dot{\phi}_X = \Big(\rho + \delta_0 + CG'(X) - \big[G'(X)F(E) + G(X)F'(E)\big]\Big)\phi_X. \tag{12}$$

[11] Note that the accounting identity, (7), implies that the choice of R follows immediately from the choice of C and the value of X that is thereby induced.

[12] Because the opportunity cost of using any installed renewable capacity is zero, and hence less than the marginal cost of oil, it can never be optimal to set $h = 0$; likewise, as marginal utility increases without bound as C approaches zero, the lower bound constraint n C can never bind. There is also an upper bound on C; in the analysis that follows, this constraint also does not bind. We therefore omit consideration of these three constraints from the pursuant discussion.

[13] The marginal cost of current consumption is the shadow value of renewable capital multiplied by the marginal effect of investment on capital accumulation, $G(X)$.

This equation of motion applies in both phases.

Finally, the solution includes a characterization of the optimal switching time, $\hat{T}$. As the stock of renewable capacity is free at this time it must be the case that the shadow value immediately before and after the switch are equal (*i.e.*, ϕ_X is continuous at $\hat{T}$). But then eq. (11) implies that C is also continuous at $\hat{T}$. In addition, the marginal effect of $\hat{T}$ upon the value functional must equal zero, which boils down to requiring

$$\phi_X(\hat{T})G(\hat{X})\Big[F\big(\hat{X}+\lambda\hat{S}\big)-F(\hat{X})\Big]-f-\gamma\lambda\hat{S}=\frac{\lambda\hat{S}D'(\hat{S})}{\rho}, \tag{13}$$

where $\hat{X}=X(\hat{T})$ and $\hat{S}=S(\hat{T})=S_0e^{-\lambda\hat{T}}$. The left side of eq. (13) is the difference between the marginal gain from slightly extending phase 1 and the extra cost that will be incurred by extracting oil for a slightly longer amount of time, while the right side is the welfare cost from slightly increasing the flow of pollution damages after the switch (by virtue of a slight increase in the pollution stock). The condition says that the net gain from slightly delaying the end of oil use just equals the marginal increase in the future discounted flow of pollution damages.

While we have simultaneous use of oil and solar during phase 1, only solar is used in phase 2. Accordingly, there are only two control variables: the rate of consumption and the rate of investment in renewable capacity. As such, this phase of the problem is analogous to a traditional neoclassical growth model.[14] We show in the appendix that the path of optimal consumption during phase 2 can be described by the equation of motion: consumption – and hence utility – will be non-decreasing if

$$\epsilon_G\frac{\dot{X}}{X}+\frac{\dot{\phi}_X}{\phi_X}\leq 0. \tag{14}$$

For a range of initial conditions, the trajectories of consumption, renewable capacity and the shadow value of renewable capacity converge to a steady state. The steady state values

[14] During phase 2 one can abstract from pollution without loss of generality. While it is true that society also bears disutility from damages associated with the pollution stock, these are independent of the controls deployed in this phase.

of consumption and renewable capacity are[15]

$$G(X^*)F'(X^*) = \rho - \delta_0(\epsilon_G - 1),$$
$$C^* = F(X^*) - \frac{\delta_0 X^*}{G(X^*)}.$$

Figure 2 provides phase diagram governing the dynamics in phase 2. We note two features of the oil-free phase. First, some minimum level of renewable capacity must be reached before society can move to the oil-free phase. This minimum level of solar capacity is a key difference with a neoclassical growth model: while oil comes at the expense of extraction and pollution costs, it helps build up the solar capacity necessary to move away from oil in the future. With insufficient oil stock, a society can be trapped in phase 1, unable to build up sufficient solar capacity. This implies declining energy use and consumption over time, down to subsistence levels as oil gets scarcer. Second, the combination of renewable capacity and consumption must be such that society enters phase 2 on a stable arm leading to the steady-state; the dynamic system therefore displays conditional stability.

4. Problem with endogenous oil production

We next consider a version of the model where oil production is chosen endogenously. As in the preceding section, herein we summarize the key elements of the solution, and relegate most of the analytic details to Section A.2 of the Appendix.

The planner's problem is to select time paths of consumption level, energy use and investment in renewable capital so as to maximize the discounted flow of net benefits over an infinite horizon. Because the energy input is the sum of non-renewable and renewable energy sources, the problem has four choice variables (C, q, h and R). Using the accounting identity on production allows us to substitute for R, leaving us with three choice variables. The decision problem is constrained by the evolution of the renewable capital stock and the

[15] The steady state rate of investment in renewable capacity may be calculated as $R^* = \delta_0 X^*/G(X^*)$.

capacity constraint on renewable usage. The problem is also influenced by the accounting identity governing output. As in section 3, output can be consumed or invested in renewable energy; in addition, the capacity constraint on renewables binds on the optimal path¿. Thus, $E = X + q$ (resp., X) in phase 1 (resp., 2). Then combining with **A3**, we obtain the equation of motion

$$\dot{X} = \begin{cases} G(X)\big(\alpha(X+q)^{\beta} - C\big) - \delta_0 X & \text{in phase 1} \\ G(X)\big(\alpha X^{\beta} - C\big) - \delta_0 X & \text{in phase 2.} \end{cases} \tag{15}$$

The planner's objective can be stated as:

$$\max_{C,h,q,\hat{T}} \left\{ \int_0^{\hat{T}} \big[U(C) - c(q) - D(P)\big] e^{-\rho t} dt + \int_{\hat{T}}^{\infty} \big[U(C) - D(\hat{P})\big] e^{-\rho t} dt \right\} \tag{16}$$

subject to **A3**, the constraints $0 \le h \le X$ and $0 \le C \le F(E)$, and where $P(t) = P_0 + S_0 - S(t)$ and $\hat{P} = P(\hat{T})$.

As in section 3, the optimal rate of consumption sets the marginal utility from consumption equal to marginal cost of current consumption (the shadow value of renewable capital multiplied by the marginal effect of investment on capital accumulation):

$$U'(C) = \phi_X G(X). \tag{17}$$

Also as before, the capacity constraint will bind, and $\nu = \phi_X G F'$. Finally, the optimal rate of oil extraction balances the marginal benefit from increased oil use against the extra cost:[16]

$$\phi_X G(X) \beta \alpha (X+q)^{\beta-1} = \gamma + \phi_S. \tag{18}$$

16 The increased oil usage raises output, which can either be consumed – generating an increase in utility – or invested – generating an increase in future value via the expansion of the renewable capacity. The additional cost is the sum of marginal extraction cost, γ, and the shadow cost associated with a reduction in the remaining stock of oil, ϕ_S.

The solution also includes equations of motion governing the two shadow values. The equation of motion for the stock of remaining oil is

$$\dot{\phi}_S = \rho\phi_S - D'(P).$$

This equation adapts a traditional Hotelling-style characterization, in which the shadow value of a non-renewable resource such as oil would appreciate at the rate of interest, by taking induced (marginal) pollution damages into account. As the capacity constraint on renewables binds, the equation of motion for the shadow value of renewable capacity is:

$$\dot{\phi}_X = \Big(\rho + \delta_0(1-\epsilon_G) - \epsilon_g \frac{\dot{X}}{X} - G(X)F'(E)\Big)\phi_X. \tag{19}$$

Finally, the solution includes a characterization of the optimal switching time, $\hat{T}$. We note first that the shadow value of remaining oil at the end of phase 1 must equal the marginal effect of the remaining oil stock upon the present discounted value of phase 2. Because the remaining oil stock decreases the pollution stock one-for-one, and it never depreciates, the marginal continuation value is

$$\phi_S(\hat{T}) = D'\big(P(\hat{T})\big)/\rho. \tag{20}$$

Because the stock of renewable capacity is free at $\hat{T}$, the current shadow value of renewable capacity immediately before and after the switch must be equal – *i.e.*, ϕ_X is continuous at $\hat{T}$. But then eq. (17) implies that C is also continuous at $\hat{T}$; moreover, $\phi_X(\hat{T})$ must equal the marginal impact of X upon the present discounted continuation value, when evaluated at $\hat{T}$. Writing $q(\hat{T}) = \hat{q}$ and employing eq. (18), this latter condition can be written as

$$\rho\Big(\frac{f+\gamma\hat{q}}{\hat{q}}\Big) + D'(P) = \alpha\Big(\rho\gamma + D'(P)\Big)\left(\frac{F(\hat{X}+\hat{q}) - F(\hat{X})}{\hat{q}F'(\hat{X}+\hat{q})}\right). \tag{21}$$

The term on the left side of eq. (21) is the difference between the marginal gain from slightly extending phase 1 (a gain) and the marginal loss from slightly delaying the start of phase 2, while the right side is the welfare cost from slightly increasing the flow of pollution damages after the switch (by virtue of a slight increase in the pollution stock). The condition says that the net gain from slightly delaying the end of oil use just equals the marginal increase in the future discounted flow of pollution damages. The implication is that society transitions – in finite time – into a phase where only renewable energy is used.[17]

The path of optimal consumption is described by the equation of motion:

$$\frac{\dot{C}}{C} = -\frac{1}{\theta}\left\{\epsilon_G \frac{\dot{X}}{X} + \frac{\dot{\phi}_X}{\phi_X}\right\}. \tag{22}$$

so that consumption (and utility) will be non-decreasing if

$$\epsilon_G \frac{\dot{X}}{X} + \frac{\dot{\phi}_X}{\phi_X} \leq 0.$$

During phase 1 we have simultaneous use of oil and solar, while only solar is used in phase 2. In phase 2 the problem consists of two control variables: the rate of consumption and the rate of investment in renewable capacity. As such, phase 2 of this problem is analogous to phase 2 of the problem considered in section 3. In particular, for a range of initial conditions, the trajectories of consumption, renewable capacity and the shadow value of renewable capacity converge to a steady state. Figure 3 depicts the time paths of renewable capacity and consumption during both phases; in this diagram both variables increase throughout time, and converge to a steady state. The steady state values of consumption, investment and renewable capacity are the same as in section2:

$$G(X^*)F'(X^*) = \rho - \delta_0(\epsilon_G - 1), \tag{23}$$

$$R^* = \delta_0 X^*/G(X^*), \tag{24}$$

$$C^* = F(X^*) - \delta_0 R^*. \tag{25}$$

[17] In this way, there is a duality between choosing the cutoff pollution stock and choosing the time of transition. The idea that there might be a cutoff level of the pollution stock is consistent with discussions at many of the annual United Nations Climate Change Conferences; it is of central importance in the Paris agreement that was negotiated in 2015. The idea has also appeared in the scientific literature, for example Allen et al. (2009). We suppose that oil is a sufficiently valuable resource that $F(\lambda S_0) > \gamma$, *i.e.*, in a scenario where only oil is used the value of output exceeds the cost of production at least for a period of time.

We note two features of phase 2. First, some minimum level of renewable capacity must be reached before society can move to the oil-free phase. This minimum level of solar capacity is a key difference with a neoclassical growth model: while oil comes at the expense of extraction and pollution costs, it helps build up the solar capacity necessary to move away from oil in the future. With insufficient oil stock, a society can be trapped in phase 1, unable to build up sufficient solar capacity. This implies declining energy use and consumption over time, down to subsistence levels as oil gets scarcer. Second, the combination of renewable capacity and consumption must be such that society enters phase 2 on a stable arm leading to the steady-state; the dynamic system therefore displays conditional stability.

As we discuss in the next section, when the initial levels of renewable capacity and the oil stock are both sufficiently small or sufficiently large, Malthusian effects – with declining standards of living – are possible.

5. Numerical Illustration

To shed further light on the dynamics of our problem, and the implications for continual growth in well-being, we produce some numerical simulation results. These simulations were based on specific parameterizations, which we believe to be non-controversial. For the production function we take $\alpha = 2, \beta = .9$. For aggregate utility, we set $\theta = .5$; with this parameterization the inter-temporal elasticity of substitution is 2.[18] We set marginal damages from the pollution stock as $d = 1$, which can be interpreted as specifying damage in terms of utils. The parameters characterizing the efficacy of investment R in renewable energy capacity X are $a = 3.5, b = \frac{a}{e}$.[19] With this specification the maximum value of G, which obtains at $X = a/b$, is equal to 1. As such, the interpretation is that efficacy is viewed as a fraction of its maximal level. Depreciation of X is taken to be $\delta_0 = 0.05$, which is also the decline rate in oil production λ. Finally, we set the rate of time preference as $\rho = 0.02$.

[18] This choice is consistent with Tahvonen and Salo (2001), who require θ to be between zero and one. In Hansen and Singleton (1982), θ ranges from 0.68 and 0.95, while van der Ploeg and Withagen (2014) assume $\theta = 2$. Setting $\theta < (>)1$ implies an intertemporal substitution greater (less) than 1.

[19] We also consider a version with a¡ 1, as discussed below.

Using this parameterization, we numerically simulated the system of differential equations during each phase.[20] The procedure was to solve for the path during phase 1, including the endogenously optimal switching time, and to then use the terminal values of X and C from phase 1 as initial conditions in phase 2, constraining the phase 2 path to converge to the steady state. Figure 3 illustrates the resultant time paths for X and C. Phase 1 levels of renewable capacity are depicted by the line with triangles, while phase 1 consumption levels are the simple line; phase 2 levels of renewable capacity are depicted by the dashed line while phase 2 consumption levels are the line with diamonds. Both variables grow slowly at the outset, reflecting the role played by the non-renewable resource early on in the program. But as this resource stock declines, society quickly switches to renewables. Oil is abandoned at time $\hat{T}$, after which consumption and renewable capacity both converge smoothly to their steady state values.

The path illustrated here is based on a moderate initial stock of nonrenewable energy. For larger initial levels, an intriguing phenomenon emerges: as in figure 3 society gradually increases renewable capacity during phase 1. Now, however, the oil stock is large enough that it takes considerably longer to reach the point in time where oil is abandoned. With this longer timeline, together with the larger levels of output that are made possible by the large initial oil stock, the renewable capital level at time $\hat{T}$ exceeds the steady state level. Accordingly, once phase 2 is entered renewable capacity and consumption both fall, traversing along the stable branch above the steady state. Figure 4 illustrates this pattern for the entire time horizon, while Figure 5 focuses on the range of time after society draws near to the switching time.

Three points emerge from this particular example. First, the time paths in this example exhibit a Malthusian effect: well-being rises for a time, but at the expense of later generations. Second, while it is apparent that utility would need to fall at some point where the initial level of renewable capacity to exceed the steady state level, this example shows non-monotonic

[20] Using the ode45 module in MATLAB, as well as the Maple computer programs.

utility can occur even if the initial level of X is not large. Third, the example reveals the significance of large initial endowments of nonrenewable energy. This result echoes Gerlagh (2010), who also finds that initial endowments of "too much oil" can ultimately be harmful. Unlike Gerlagh, our result emerges not because of overly large pollution damages, but because society is lured into excessive development of a key capital stock. This feature is reminiscent of the overshooting effect found in van der Ploeg and Withagen (2014).[21]

It is also possible for Malthusian-like effects to arise when the initial stocks of oil and renewable capacity are small. Figure 6 illustrates such an example. This example is built up from a smaller value of a $(= 0.75)$, along with a larger value of b $(= 8.4075)$; we also change α, now setting it equal to 1. All other parameters are as above. With this parameterization there are two steady states. The one with larger consumption and larger renewable capacity is saddlepoint stable, so there are paths converging to that steady state; figure 7 illustrates. There, the initial stocks of oil and renewable capacity are sufficient to allow production levels that support trajectories with ever-rising utility. By contrast, the steady state with smaller consumption and smaller renewable capacity is unstable. With two steady states there are combinations of initial stock levels that lie outside of (below) the basin of attraction for the stable steady state. As such, these initial conditions can not support trajectories that lead to the good steady state – instead, they converge asymptotically to the origin. The interpretation here would be that of an impoverished economy, which is doomed to a deeply pessimistic future. While the underlying reasons differ from Malthus' original narrative, they are spiritually similar.

6. Decentralized problem

In this section we sketch a decentralized model, and compare it the the (first best) solution to the social planner's problem analyzed above. We adopt assumptions conventionally

[21] van der Ploeg and Withagen (2012) are also skeptical of the "too much oil" story, but for a very different reason: they argue that the important concern is that there is too much of a much dirtier resource (coal).

employed in the literature.[22] A representative consumer maximizes utility that is tied to a consumption good C, via $U(C)$. This good is supplied by a perfectly competitive industry. The production of this good is based on energy, using the production function in **A1**. The market price paid by consumers equals marginal utility $U'(C)$; this in turn equals marginal cost, which is the ratio of the price of energy P to its marginal product $F'(E)$.

Energy is supplied by an industry with N firms, all of whom have access to both types of energy resources (*i.e.*, oil and solar). To streamline the presentation we retain the assumption that oil production follows a decline curve, and assume that all firms have initial oil deposits of size s_0; thus, firm j supplies energy

$$e_j = \lambda s_0 + h_j,$$

where $h_j \leq x_j$ is j's supply of renewable energy, and x_j is its renewable capacity. Extraction of oil entails a flow fixed cost $\tilde{\gamma}$. The evolution of the firm's renewable capacity is given by

$$\dot{x}_j = G(X) r_j - \delta x_j,$$

where r_j is investment rate in renewable capacity undertaken by the firm. As in the planner's problem, the efficacy of this investment depends on a function G, which captures both learning by doing and Ricardian effects; for comparability with the earlier discussion we retain the specific form

$$G(X) = X^a e^{-bX}$$

given in eq. (1).

The relation between X and G deserves comment. The Ricardian effects we described above would surely depend on industry development (and hence land acquisition), which explains the presence of X in the exponential term. And while one might envision the

[22] See, for example, van der Ploeg and Withagen (2012); André and Smulders (2014) and Golosov et al. (2014). Tsur and Zemel (2011) also analyze a decentralized equilibrium, but do not account for the disutility from pollution arising from extracting non-renewable resource, nor do they include learning-by-doing or Ricardian effects in their model.

learning-by-doing effects resulting the firm's individual experience, we assume the dominant effect arises from industry experience. For example, this seems to be the case with solar, where installation costs of new capacity have fallen over time on the basis of experience acquired by installers. Under this interpretation, the efficacy of investment monies depend on the combined historical experience of all firms, which is summarized by total industry capacity X; this explains the presence of X in the power function component of G.

The typical firm's current-value Hamiltonian in phase 1 is

$$\mathcal{H}_j = P\big(\lambda s_0 + h_j\big) - \tilde{\gamma} - r_j + \varphi\big(G(X)r_j - \delta_0 x_j\big) + \nu_j(x_j - h_j). \tag{26}$$

This leads to the optimization rules:

$$\frac{\partial \mathcal{H}_j}{\partial h_j} P - \nu_j \text{ if } h_j > 0, \tag{27}$$

$$\frac{\partial \mathcal{H}_j}{\partial r_j} = -1 + \varphi x_j G(X), \tag{28}$$

$$\dot{\varphi}_j = \rho\varphi - \frac{\partial \mathcal{H}_j}{\partial x_j} = (\rho + \delta_0 - r_j \frac{\partial G}{\partial x_j})\varphi - \nu_j, \tag{29}$$

$$\nu_j \geq 0, x_j \geq h_j, \nu_j(x_j - h_j) = 0. \tag{30}$$

As above, it is straightforward to show that combining the optimality condition on h with the complementary slackness condition, eqs. (26) and (29), leads to the conclusion that $h_j = x_j$. It then follows that

$$\nu_j = P.$$

Finally, the firm ceases oil extraction at that moment $\tilde{T}$ when flow profits have fallen to zero, or

$$P\lambda s_0 e^{-\lambda \tilde{T}} = \tilde{\gamma}.$$

With N identical firms, total (industry) extraction costs are $N\tilde{\gamma}$, which corresponds to γ in the planner's problem; total initial reserves are $S_0 = N s_0$. It follows that the switching time in the decentralized model can be expressed as

$$\tilde{T} = \frac{ln(Ps_0/\tilde{\gamma})}{\lambda},$$

which is unambiguously larger than $\hat{T}$ in the planner's problem. This difference comes down to the fact that firms in the decentralized problem do not take into account the effects their actions have on social damages, via the pollution stock – *i.e.*, that their actions induce a negative externality.

To further develop the discussion, we need to elaborate on the impact of a small change in the firm's capacity upon G. If one takes the view that the firm is atomistic, *i.e.*, N is very large, then presumably $\dfrac{\partial G}{\partial x_j} = 0$. Alternatively, one might imagine the firm treats all other firms' capacity profiles as fixed, so that $\dfrac{\partial G}{\partial x_j} = G'$. We take the latter approach below. With this assumption, the equation of motion for the firm's shadow price of renewable capacity can be reduced to

$$\begin{aligned}\dot{\varphi}_j &= (\rho + \delta_0 - r_j)\varphi - P \\ &= (\rho + \delta_0 - \frac{R}{N}G')\varphi - U'F' \end{aligned} \tag{31}$$

as there are N identical firms, and the competitive equilibrium in the consumption good market ensures $U' = p/F'$. By comparison, the equation of motion for the shadow value of renewable capacity in the socially optimal program is, from eq. (12)

$$\dot{\phi}_X = (\rho + \delta_0)\phi - \left[\epsilon_G(\frac{\dot{X}}{X} + \delta_0) - GF'\right]\phi_X.$$

Using **A3**, eq. (17) and the definition of ϵ_G, this expression can be simplified to

$$\dot{\phi}_X = (\rho + \delta_0 - RG')\phi_X - U'F'. \tag{32}$$

Comparing eqs. (30) and (31), we see the difference comes down to a comparison between RG' and $\frac{R}{N}G'$. For $N > 1$ these can not be equal, which is intuitive: because learning exhibits spillover effects, there is a positive externality that is not captured in the private equilibrium. As we noted above, there is also a difference between the decentralized and socially optimal switching times, which reflects the negative externality firms' oil use imposes on society. To align the private and socially optimal equilibria then would require the use of two instruments: a pollution tax (or cap on ultimate extraction), and a subsidy for investment in renewable capacity.[23]

7. Problem with two inputs into production

We now turn to the problem when both man-made capital (K) and energy influence aggregate production. As with the first variant, we give a brief discussion of the solution here, and delegate most of the analytic details to Appendix B. In this variant of the problem, there are two types of investment (I and R), as well as a new state variable. We set the equation of motion for the stock of man-made capital as:

$$\dot{K} = I - \delta_K K, \tag{33}$$

i.e., capital increases with investments I and depreciates at rate δ_K. The accounting identity governing consumption and investments becomes

$$F(E, K) = C + I + R + \gamma,$$

where the last term represents extraction costs (and hence only applies if extraction is positive). As in section 3 there is a constraint on pollution, which induces a cutoff level $\hat{S}$ on the remaining stock of oil – so that there will be two phases, one where both types of energy are used and one where only renewable energy is used.

[23] Compare with Acemoglu et al. (2012), who also finds that these two policy tools are required to induce a first-best outcome in a decentralized framework. Subsidizing clean energy and investing capacity expansion lead to a weak Green Paradox (Sinn, 2008; van der Ploeg and Withagen, 2015). A strong Green Paradox is found to occur when renewables capacity is subsidized, as it gives an incentives to pollute before a switch to renewable energy occurs.

This variant is a bit easier to work with when posed as a calculus of variations problem. and so we adopt that approach. To that end, it is most convenient to rewrite the accounting identity so as to isolate consumption:

$$C = F(E, K) - I - R, \tag{34}$$

and write utility in terms of the right-hand side of eq. (33).

The solution to this variant of the problem is characterized by the Euler equations, of which there are two; in this setting, it will be convenient to work with the ratio of the two capital stocks:

$$\chi = \frac{X}{K}.$$

Using this definition, the Euler equations can be written as

$$\theta \frac{\dot{C}}{C} = \alpha(1-\beta)\chi^{\beta} - (\rho + \delta_K); \tag{35}$$

$$\theta \frac{\dot{C}}{C} = \alpha\beta\chi^{\beta-1}G(X) + \big(\delta_0\epsilon_G - 1\big)\Big(\frac{\dot{X}}{X} + 1\Big) - \rho. \tag{36}$$

These conditions can be combined to yield a first-order differential equation that governs the solution:

$$\alpha\chi^{\beta-1}\big(\beta G(X) - (1-\beta)\chi\big) + \big(\delta_0\epsilon_G - 1\big)\Big(\frac{\dot{X}}{X} + 1\Big) - \delta_K = 0.$$

At the steady state for this system $\dot{C} = \dot{X} = 0$; using eqs. (34)–(35) we have

$$(\chi^*)^{\beta} = \frac{\rho + \delta_K}{\alpha(1-\beta)}, \tag{37}$$

$$(\chi^*)^{(\beta-1)} = \frac{\rho + 1 - \delta_0(a + bX^*)}{\alpha\beta G(X^*)}. \tag{38}$$

As in section 3, the optimal program can be split into two phases: both oil and solar are used in phase 1, but only solar is used in phase 2. As above, the system may converge to a steady state in phase 2; we illustrate some possible paths in figure 8. This diagram shows the stable trajectories for utility U, capital K and the renewable resource stock X for five scenarios; the steady-state is represented as the black dot in the centre. The trajectory identified by circles shows the case for which both $X(0)$ and $K(0)$ are each half their steady-state value, which we refer to as "scenario (i)" in the discussion below. Figure 8 also includes trajectories corresponding to the following scenarios: scenario (ii) sets $K(0) = 0.5K^*$ and $X(0) = 1.5\ X^*$; scenario (iii) sets $K(0) = 1.5K^*$ and $X(0) = 0.5X^*$; scenario (iv) sets $K(0) = 1.5K^*$ and $X(0) = 1.5X^*$.

As the figure illustrates, any trajectory starting from initial values of X and K that are larger than their steady-state value would imply that utility must decline at some point. Thus, only scenario (i) is sustainable; the other scenarios in the Figure are not sustainable. Thus, a necessary condition for sustainability is that the initial values of X and K are smaller than the corresponding steady-states. But there are trajectories with a small initial value of K that is not sustainable, so the condition is not sufficient.

The figure also illustrates a trajectory (v) that temporarily prevents investment in renewable capacity. This scenario can be interpreted as one in which society has placed a moratorium on development of renewable energy, which could be thought of as a manifestation of the energy policies undertaken by the Trump administration in the US over the past few years. We see that this policy delivers an unsustainable path, inasumch as it promotes large consumption levels in the short run. Because the levels of X and K cannot be instantaneously adjusted, society has to immediately lower consumption, which causes a one-time loss in utility – as represented by the arrow on the graph.

8. Concluding remarks

The existing economics literature on sustainability and energy use typically assumes that energy from renewable natural capital can be bought at a fixed price, with no other constraint. This assumption is quite strong, particularly from the social planner's standpoint, as it ignores the dynamic aspects of renewable natural capital and its accumulation. It also commonly leads to an empirical prediction that is at odds with reality: that society will use the non-renewable energy resource exclusively up to a certain time, at which point it will switch to using renewable energy exclusively. In our view, the most natural way to extend the analysis so as to circumvent this empirical inconsistency, is to impose limits on the magnitude of renewable energy use in accordance with a capacity constraint. This constraint arises from a capital stock that facilitates the exploitation of renewable energy. With this interpretation, for society to reap the benefits from the renewable resource it has to continually invest: While delaying investment yields a short increase in welfare, this increase dissipates quickly. In addition to producing a more empirically satisfactory explanation for simultaneous use of non-renewable and renewable energy, we also find that it can be optimal for society to cease use of non-renewable energy, switching to the exclusive use of renewable energy, even though the non-renewable resource stock is not fully exhausted.[24] Our paper provides such an extension.

It is optimal to completely switch from non-renewable to renewable resources in energy production, and in finite time. Interpreting sustainability as the restriction that utility never decrease as time goes by, we find find paths that are sustainable. Sustainability can arise when the initial levels of the relevant state variables are neither too large, nor too small. Certainly they need to be smaller than their respective steady-state values. Any trajectory that requires a reduction in one of the state variables is unsustainable, as is any scenario that requires society accept a reduction in output, for example when a moratorium on additions to renewable capacity is imposed. But Malthusian-like paths can also emerge when society is endowed with a large initial stock of the non-renewable resource.

Important policy implications emerge from this framework. Although we did not discuss the manner in which society determines the allocation of resource bases to energy production, we can envision a number of approaches. Society could impose standards that require a certain level of usage of renewables, as with Renewable Performance Standards – popular

[24] This calls to mind the quote from Sheik Yamani, former oil minister for OPEC: "[t]he stone age did not end for lack of rocks, and the oil age will not end for lack of oil."

in some US states. Or society could adopt a tradable permit scheme for carbon emissions, as with the EU's emission trading system. A third option is to invoke a carbon tax – as in the Canadian province of British Columbia – which raises the cost of (non-renewable) fossil fuels. The presence of a capacity constraint on the use of renewables would suggest that none of these policies can induce increased renewable production in the moment (as the capacity constraint would preclude such expansion), though it seems likely to encourage increased investment in renewable capacity – and perhaps research into new innovations (Acemoglu et al., 2012). In this regard, the financial rewards associated with avoiding the the carbon tax would seem pose an attractive incentive in a decentralized economy, such as that found in most Western countries.

As with any model, we have imposed a structure that invites extension. For example, we assumed perfect substitution between renewable and non-renewable resources in energy generation. In practice, renewable energies are subject to intermittency and storage constraints which can inhibit substitution (Pommeret and Schubert, 2018). That said, one would expect that it is possible to produce some energy even if $q = 0$. For it to be possible to generate energy if society no longer relies on non-renewable resources, *i.e.*, $E(0, h) > 0$, one of two scenarios would obtain: Either non-renewable capital would be exhausted in finite time, in which case society would reach a post non-renewable resource phase similar to the one described above; or society would have to manage its use of both types of natural capital so as to asymptotically converge to the post non-renewable resource steady-state. In the second situation, non-renewable resources would be gradually phased out in favor of their renewable alternative. Thus, the steady-state values of physical and renewable natural capital, consumption and energy shares correspond to post non-renewable regime we discussed above, whether society switches to renewable energies in finite time or infinite time (Vardar, 2013). Given the dynamic nature of energy investments, any new policy aiming at promoting renewable energies must carefully examine the possibility and cost associated 'dirty' assets being stranded following policy implementation. Stranded assets can arise from inconsistent policy signals following political elections or from the possibility for some elected officials to veto policies announced by other elected officials. Such time-inconsistencies are costly when investments in dirty assets are irreversible; accounting for them changes investment patterns (Kalkuhl et al., 2020). Indeed, some authors have suggested that we should switch to investing solely in renewables so as to constrain the rise in temperature to 2°C (Baldwin et al., 2020). Finally, we have abstracted from the use of abatement to reduce the pollution stock. Such an extension would enhance the realism of the structure, but at the cost of adding an additional control variable. While such an addition would refine the analysis, it seems unlikely to change the spirit of the analysis, nor our central results.

Appendix A. Technical details of the planner's problem, variant 1

In this Appendix, we provide the technical details for the models in section 3 and section 4.

A.1. **Exogenous oil.** The planner's problem is to maximize the present discounted flow of utility less pollution damages. Damages are

$$\begin{aligned}
\Delta &= \int_0^\infty \frac{d}{2}\left(P_0 + \kappa(S_0 - S(t))\right)^2 e^{-\rho t} dt \\
&= \int_0^{\hat{T}} \frac{d}{2}\left(P_0 + \kappa(S_0 - S(t))\right)^2 e^{-\rho t} dt + \int_{\hat{T}}^\infty \frac{d}{2}\left(P_0 + \kappa(S_0 - S(\hat{T}))\right)^2 e^{-\rho t} dt \\
&= \int_0^{\hat{T}} \frac{d}{2}\left(P_0 + \kappa(S_0 - S(t))\right)^2 e^{-\rho t} dt + \frac{d}{2}\left(P_0 + \kappa(S_0 - S(\hat{T}))\right)^2 \frac{e^{-\rho \hat{T}}}{\rho} dt.
\end{aligned}$$

The present discounted value of the flow of utility can be expressed as $I_1 + I_2$, where

$$\begin{aligned}
I_1 &= \int_0^{\hat{T}} U(C) e^{-\rho t} dt \\
I_2 &= \int_{\hat{T}}^\infty U(C) e^{-\rho t} dt.
\end{aligned}$$

The value functional is then the present discounted value of the flow of utility, less the present discounted value of extraction costs, less damages:

$$\mathcal{V} = I_1 + I_2 - \Delta.$$

$$\max_{C,h,\hat{T}} \left\{ \int_0^\infty U(C)e^{-\rho t}dt - \int_0^{\hat{T}} \Big[c\big(q(t)\big) + D\big(P(t)\big)\Big]e^{-\rho t}dt - \int_{\hat{T}}^\infty D\big(P(\hat{T})\big)e^{-\rho t}dt \right\}. \quad \text{(A.1)}$$

which is eq. (10) in the main text. One can regard this dynamic optimization problem as a conventional optimal growth model, augmented by a salvage value; in such an interpretation the salvage value is

$$\omega(\hat{T}) = -\Big(\int_0^{\hat{T}} \Big[c\big(q(t)\big) + D\big(P(t)\big)\Big]e^{-\rho t}dt + \int_{\hat{T}}^\infty D\big(P(\hat{T})\big)e^{-\rho t}dt\Big). \quad \text{(A.2)}$$

To describe the solution to this problem, we first form $\mathscr{H}_i$, the current-value Hamiltonian-Lagrangean for phase $i = 1, 2$. To this end, we note first that because the opportunity cost of using any installed renewable capacity is zero (and hence less than the marginal cost of oil), it can never be optimal to set $h = 0$; likewise, as marginal utility increases without bound as C approaches zero, the lower bound constraint on C can never bind. There is also an upper bound on C; in the analysis that follows, this constraint also does not bind. We therefore omit consideration of these three constraints from the pursuant discussion, and so:

$$\mathscr{H}_1 = U(C) + \phi_X\big[G(X)\big(F(h + S_0e^{-\lambda t}) - C\big) - \delta_0 X\big] + \nu(X - h), \quad \text{(A.3)}$$

$$\mathscr{H}_2 = U(C) + \phi_X\big[G(X)\big(F(h) - C\big) - \delta_0 X\big] + \nu(X - h). \quad \text{(A.4)}$$

The solution to the optimization problem satisfies Pontryagin's maximum principle:

$$0 = \frac{\partial \mathscr{H}}{\partial C} \iff U'(C) = \phi_X G(X); \quad \text{(A.5)}$$

$$\dot{\phi}_X = \rho\phi_X - \frac{\partial \mathscr{H}}{\partial X} = \Big(\rho + \delta_0 + CG'(X) - \big[G'(X)F(E) + G(X)F'(E)\big]\Big)\phi_X; \quad \text{(A.6)}$$

as well as the complementary-slackness conditions

$$\frac{\partial \mathscr{H}}{\partial h} = \phi_X G(X) F'(E) - \nu \leq 0, h \geq 0, h(\phi_X G(X) F'(E) - \nu) = 0, \tag{A.7}$$

$$\frac{\partial \mathscr{H}}{\partial \nu} = X - h \geq 0, \nu \geq 0, \nu(X - h) = 0. \tag{A.8}$$

These conditions imply the capacity constraint must bind. Suppose to the contrary that the constraint did not bind at some time, so $h < X$. Then the last part of (A.8) implies $\nu = 0$ at that moment. But ϕ_X, G and E' are all positive, in which case $\nu = 0$ would imply $\phi_X G(X) F'(E) - \nu > 0$, in violation of the first part of the complementary slackness condition (A.8). We conclude the original supposition was false, and conclude that the capacity constraint binds at all times.[25]

Finally, the solution includes a characterization of the optimal switching time, $\hat{T}$. Viewing this as a free end-time problem for phase 1, combined with a free start-time problem for phase 2, and applying the transversality conditions from Léonard and van Long (1992), we find

$$\mathscr{H}_1 - \mathscr{H}_2 + e^{\rho \hat{T}} \omega'(\hat{T}) = 0.$$

From eq. (A.2) we have

$$\begin{aligned} \omega'(\hat{T}) &= -c\big(q(\hat{T})\big) e^{-\rho \hat{T}} - D'\big(P(\hat{T})\big) P'(\hat{T}) \int_{\hat{T}}^{\infty} e^{-\rho t} dt \\ &= -e^{-\rho \hat{T}} \left(f + \gamma \lambda S(\hat{T})\big) + D'\big(P(\hat{T})\big) P'(\hat{T}) \right) \end{aligned} \tag{A.9}$$

Then combining eqs. (A.3), (A.4) and (A.9) and rearranging, we obtain

$$\phi_X(\hat{T}) G(\hat{X}) \Big[F(\hat{X} + \lambda S_0 e^{-\lambda \hat{T}}) - F(\hat{X}) \Big] - f - \gamma \lambda S_0 e^{-\lambda \hat{T}} = \frac{dP(\hat{T}) \kappa \lambda S_0 e^{-\lambda \hat{T}}}{\rho}. \tag{A.10}$$

A.2. **Endogenous oil.** We now discuss the extension to the basic model, wherein oil use is not constrained by decline curve effects. In this variant, the rate of oil use is an additional choice variable, the remaining stock is an additional state variable, and there is an additional

[25] In addition, the first part of (A.8) yields $\nu = \phi_X G F'$.

co-state variable ϕ_S (the shadow value of remaining oil). The first thing to note is that behavior in phase 2 is essentially unchanged by this extension – the steady state is as before, and the path dynamics are still described by equations of motion for C and X. Behavior in phase 1 is potentially changed, and so our discussion is focused there.

$$\max_{C,h,\hat{T}} \left\{ \int_0^\infty \Big(U(C) - D(P) - c(q) \Big) e^{-\rho t} dt \right\} \tag{A.11}$$

$$\begin{aligned} \mathscr{H} =& U(C) - D(P_0 + \kappa(S_0 - S(t))) - c(q) + \phi_X \big[G(X) \big(F(q) - C \big) - \delta_0 X \big] \\ &+ \phi_S(-q) + \nu(X - h), \end{aligned} \tag{A.12}$$

The solution to the optimization problem satisfies Pontryagin's maximum principle:

$$0 = \frac{\partial \mathscr{H}}{\partial C} \iff U'(C) = \phi_X G(X); \tag{A.13}$$

$$\dot{\phi}_X = \rho\phi_X - \frac{\partial \mathscr{H}}{\partial X} = \Big(\rho + \delta_0 + CG'(X) - \big[G'(X)F(E) + G(X)F'(E) \big] \Big) \phi_X; \tag{A.14}$$

$$\dot{\phi}_S = \rho\phi_S - \frac{\partial \mathscr{H}}{\partial X} = \rho\phi_S - \kappa D'(P(t)); \tag{A.15}$$

as well as the complementary-slackness conditions

$$\frac{\partial \mathscr{H}}{\partial h} = \phi_X G(X) F'(E) - \nu \leq 0, h \geq 0, h(\phi_X G(X) F'(E) - \nu) = 0, \tag{A.16}$$

$$\frac{\partial \mathscr{H}}{\partial \nu} = X - h \geq 0, \nu \geq 0, \nu(X - h) = 0, \tag{A.17}$$

$$\frac{\partial \mathscr{H}}{\partial q} = \phi_X G(X) F'(E) - c'(q) - \phi_S \leq 0, q \geq 0, q(\phi_X G(X) F'(E) - c'(q) - \phi_S) = 0, \tag{A.18}$$

$$\tag{A.19}$$

The objective functional for phase 1 is now:

$$\int_0^{\hat{T}} \big(U(C) + dS \big) e^{-\rho t} dt,$$

with associated current value Hamiltonian[26]

$$\mathcal{H}_1 = U(C) - dS + \phi_X \Big[G(X) \big[F(X+q) - C - \gamma \big] - \delta_0 X \Big] - (1+\mu) q \phi_S + \nu (X - h).$$

The optimality conditions for C and $\dot{\phi}_X$ are as in section 3 above, but Pontryagin's maximum principle is now augmented by the inclusion of two new equations:

$$\begin{aligned} 0 &= \frac{\partial \mathcal{H}_1}{\partial q} = \phi_X G(X) F'(X+q) - (1+\mu)\phi_S, \\ \dot{\phi}_S &= \rho \phi_S - \frac{\partial \mathcal{H}_1}{\partial S} = \rho \phi_S + d. \end{aligned}$$

The solution to the equation of motion for ϕ_S is easily seen to be

$$\phi_S = \phi_S^0 e^{\rho t} + \frac{d}{\rho}.$$

Making use of eq. (17), we rewrite the optimality condition for q as

$$U'(C) F'(X+q) = (1+\mu) \phi_S^0 e^{\rho t} + \frac{d(1+\mu)}{\rho}. \tag{A.20}$$

The left-hand side of eq. (A.20) is the marginal value product of oil in utils, while the first term on the right represents the resource-related opportunity cost of extraction; accordingly, this equation can be interpreted as a modified Hotelling rule, where the modification arises from the second term on the right-hand side. This second term is related to the social cost of carbon.

The solution is completed by determining the optimal date to cease extraction of oil. But this is dictated by the same rule as in the basic model, namely that the difference in the Hamiltonian just before and just after $\hat{T}$ equals the marginal impact of a slight delay in switching upon the future present discounted flow of damages implied by a slight increase in the pollution stock.

[26] There is also a non-negativity constraint on q, but that is moot since by definition $q > 0$ in phase 1.

A.2.1. *Dynamics of solution.* Starting from the optimality condition for C, eq. (17), we time differentiate both sides to obtain an equation of motion for consumption:

$$U''(C)\dot{C} = \phi_X G'(X)\dot{X} + \dot{\phi}_X G(X)$$
$$= \phi_X G(X)\left\{\epsilon_G \frac{\dot{X}}{X} + \frac{\dot{\phi}_X}{\phi_X}\right\}. \tag{A.21}$$

Using **A3**, eq. (A.21) then implies

$$\frac{\dot{C}}{C} = -\frac{1}{\theta}\left\{\epsilon_G \frac{\dot{X}}{X} + \frac{\dot{\phi}_X}{\phi_X}\right\}.$$

which is eq. (**??**) in the text. In light of eq. (19), we observe that this expression can be rewritten as

$$\frac{\dot{C}}{C} = -\frac{1}{\theta}\left\{\rho + \delta_0(1-\epsilon_G) - GF'\right\}. \tag{A.22}$$

These dynamics apply whether we are in the phase where both oil and solar are used (phase 1), or only solar is used (phase 2). For consumption – and hence utility – to grow all along the path from X_0 to X^*, the term inside the brackets on the right-hand side of eq. (A.21) must be negative, implying

$$G(X)F'(X) - \delta_0 bX > \rho + \delta_0(1-a), \forall X \in [X_0, X^*]. \tag{A.23}$$

We conclude this sub-section by constructing the second-order differential equation in X that governs the solution. Combining the accounting identity with **A3**, we obtain an equation of motion governing X. Recalling that $E = X + \lambda S_0 e^{-\lambda t}$ (resp. X) in phase 1 (resp. 2), we have

$$\dot{X} = \begin{cases} G(X)\big(F(E) - C - \gamma\big) - \delta_0 X & \text{in phase 1,} \\ G(X)\big(F(E) - C\big) - \delta_0 X & \text{in phase 2.} \end{cases} \tag{A.24}$$

Time-differentiating both sides than gives the second-order differential equation

$$\ddot{X} = \begin{cases} G'(X)\big(F(E) - C - \gamma\big)\dot{X} + F'G\dot{E} - G\dot{C} - \delta_0\dot{X} & \text{in phase 1,} \\ G'F\dot{X} + F'G\dot{E} - G\dot{C} - \delta_0\dot{X} & \text{in phase 2.} \end{cases} \tag{A.25}$$

We observe that

$$\dot{E} = \begin{cases} \dot{X} - \lambda^2 S_0 e^{-\lambda t} & \text{in phase 1,} \\ \dot{X} & \text{in phase 2.} \end{cases} \tag{A.26}$$

Combining the accounting identity with eq. (A.22) we have

$$-G\dot{C} = \begin{cases} \big[FG - \gamma G - \dot{X} - \delta_0 X\big]\big[\rho + \delta_0(1 - a + bX) - GF'\big] & \text{in phase 1,} \\ \big[FG - \dot{X} - \delta_0 X\big]\big[\rho + \delta_0(1 - a + bX) - GF'\big] & \text{in phase 2.} \end{cases}$$

The remaining terms can be synthesized after using (A.24) to substitute for C, and combining terms; this process yields

$$G'(X)\Big(F(E) - C - \gamma\Big)\dot{X} + F'G\dot{E} - \delta_0\dot{X} = \left(\epsilon_G \frac{GF}{X} - \delta_0\right)\dot{X} - \lambda\beta\big(E - X\big)\frac{GF}{E}$$

in phase 1, and

$$G'(X)\big(F(E) - C\big)\dot{X} + F'G\dot{E} - \delta_0\dot{X} = \big(G'F + \delta_0(\epsilon_G - 1)\big)\dot{X}$$

in phase 2.

Combining these observations, we obtain a second-order differential equation for X, the solution to which describes the evolution of our system. That differential equation is

$$\begin{aligned} &\ddot{X} - \left(\epsilon_G \frac{GF}{X} - \delta_0\right)\dot{X} + \lambda\beta\big(E - X\big)\frac{GF}{E} \\ &-\big[FG - \gamma\lambda G - \dot{X} - \delta_0 X\big]\big[\rho + \delta_0(1 - a + bX) - GF'\big] = 0 \end{aligned} \tag{A.27}$$

in phase 1, and

$$\ddot{X} - \Big(G'F + \delta_0(\epsilon_G - 1)\Big)\dot{X} - \big[FG - \dot{X} - \delta_0 X\big]\big[\rho + \delta_0(1 - a + bX) - GF'\big] = 0 \quad \text{(A.28)}$$

in phase 2. The solution to the second-order differential equation governing X is fully determined by two boundary conditions. In phase 1 these conditions are $X(0) = X_0, X(\hat{T}) = \hat{X}$. Assuming the system converges to a steady state in phase 2, with steady state renewable capacity level X^*, these boundary conditions for phase are $X(\hat{T}) = \hat{X}, lim_{T\to\infty}X(T) = X^*$.

A.2.2. *Steady state.* The next question to be addressed is whether a long-run equilibrium – *i.e.*, a steady state – exists. Any steady state would obtain in phase 2, and would be defined by the solution to the following two equations:[27]

$$\dot{X} = 0;$$

$$\dot{\phi}_X = 0.$$

Define the function:

$$\Omega(X) = G(X)F'(X) + (\epsilon_G - 1)\delta_0 - \rho \quad \text{(A.29)}$$

Using eq. (19), noting that $E = X$ in phase 2, and setting $\dot{X} = 0$, the $\dot{\phi}_X = 0$ condition can be written as:

$$\Omega(X^*) = 0 \Leftrightarrow$$

$$G(X^*)F'(X^*) = \rho - \delta_0(\epsilon_G - 1),$$

which is eq. (22) in the text. We note that using **A1** and eq. (1), X^* can be implicitly defined by:

$$\alpha\beta(X^*)^{a+\beta-1}e^{-bX^*} - \delta_0 bX^* = \rho + \delta_0(1 - a). \quad \text{(A.30)}$$

27 These two conditions, combined with eq. (**??**), imply $\dot{C} = 0$.

Using **A3**, the $\dot{X} = 0$ condition can be written as:

$$G(X^*)R^* - \delta_0 X^* = 0 \Leftrightarrow$$
$$R^* = \delta_0 X^*/G(X^*),$$

which is eq. (23) in the text. Combining this with the accounting identity, we may describe the $\dot{X} = 0$ iso-cline by

$$C(X) = F(X) - \delta_0 \frac{X}{G(X)}; \tag{A.31}$$

the steady state consumption level can then be determined as $C^* = C(X^*)$:

$$C^* = F(X^*) - R^*,$$

which is eq. (24) in the text. This expression can be expanded as

$$C^* = \alpha(X^*)^\beta - \delta_0 X^*/G(X^*). \tag{A.32}$$

It is easy to see that $C(X)$ goes to $-\infty$ as X tends to 0 or ∞. Moreover, we may calculate

$$\begin{aligned} C'(X) &= F' - \frac{\delta_0}{G} + \frac{\delta_0 X G'}{G^2} \\ &= F' + \frac{\delta_0}{G}(\epsilon_G - 1). \end{aligned} \tag{A.33}$$

We observe that there is a value $\check{X} > \frac{a-1}{b}$ such that $C'(\check{X}) = 0$, with $C' >$ (resp., $<$) 0 as $X <$ (resp., $>$) $\check{X}$; *i.e.*, $C(X)$ is concave. Combining eqs. (22) and (A.33), we see that

$$C'(X^*) = F'(X^*) + \frac{\rho - G(X^*)F'(X^*)}{G(X^*)} = \frac{\rho}{G} > 0.$$

Accordingly, $X^* < \check{X}$: the steady state lies to the left of the peak in the $\dot{X} = 0$ iso-cline.

We now study the general shape of the phase diagram, including the $\dot{C} = 0$ and $\dot{X} = 0$ iso-clines. These are defined by eqs. (A.30) and (A.31), respectively. Since eq. (A.30) determines X^* independent of C, the $\dot{C} = 0$ iso-cline is a vertical line at X^*. From the discussion above, we know that $\dot{C} >$ (resp., $<$) for $X <$ (resp., $>$) X^*. Referring back to eq. (A.25), it is clear that $\dot{X}$ is decreasing in increasing C, and so $\dot{X} <$ (resp., $>$) 0 at points above (resp., below) the $\dot{X} = 0$ iso-cline.

Of course, moving away from the $\dot{X}$ iso-cline might also influence $\dot{C}$. To study the comparative statics and the dynamics of the system, we linearize around the steady state and then evaluate the Jacobian matrix associated with eqs. (A.30) and (A.32). When evaluated at steady-state, the Jacobian Matrix is

$$M = \begin{pmatrix} \partial \dot{C}/\partial C & \partial \dot{C}/\partial X \\ \partial \dot{X}/\partial C & \partial \dot{X}/\partial X \end{pmatrix}\Bigg|_{(C^*,X^*)} = \begin{pmatrix} 0 & \frac{C^*}{\theta}\left(\frac{dG(X^*)F'(X^*)}{dX} - b\delta_0\right) \\ -G(X^*) & \rho \end{pmatrix}. \tag{A.34}$$

The determinant of this matrix is

$$\begin{aligned} \det(M) &= \frac{C^* G(X^*)}{\theta}\left(\frac{dG(X^*)F'(X^*)}{dX} - \delta_0 b\right) \\ &\propto \frac{dG(X^*)F'(X^*)}{dX} - \delta_0 b. \end{aligned} \tag{A.35}$$

As we noted above, $\frac{dG(X^*)F'(X^*)}{dX} < 0$ for $X > \frac{a}{b} - \frac{1-\beta}{b}$.

The characteristic polynomial for this matrix is

$$\theta\xi^2 - \rho\theta\xi + C^* G(X^*)\Omega_X,$$

where $\Omega_X = \partial \dot{C}/\partial X|_{C^*,X^*}$, with associated eigenvalues

$$\xi = \frac{\rho}{2} \pm \frac{\sqrt{\theta^2\rho^2 - 4\theta C^* G(X^*)\Omega_X}}{2\theta}.$$

Since Ω_X is negative at steady-state, one eigenvalue is positive and the other is negative, implying the steady-state is saddle-path stable. This impacts phase 1 dynamics, as a smooth transition to phase 2 implies that some solar capacity has been built prior to switching.

Appendix B. Technical details of the planner's problem, variant 2

In this appendix, we provide the technical details for the model in section 4. The approach we use here is calculus of variations, which is based on a characterization of the integrand in the optimization problem. Here, that integrand is the discounted difference between utility and pollution damages; as the latter is a constraint in our application we abstract from it. To that end, we define

$$\begin{aligned}\mathcal{F} &= e^{-\rho t}U(C)\\ &= e^{-\rho t}U(F(K,X) - R - I),\end{aligned}$$

where the production function is given by **A2** , the utility function is given by **A3**, $R = \frac{\dot{X}+\delta_0 X}{G(X)}$ (from **A4**) and $I = \dot{K} + \delta_K K$ (from eq. (32). Making these substitutions, we obtain

$$\mathcal{F} = e^{-\rho t}U\Bigg(\alpha X^{\beta}K^{1-\beta} - \left(\frac{\dot{X}+\delta_0 X}{G(X)}\right) - (\dot{K} + \delta_K K)\Bigg). \tag{B.1}$$

The solution to this problem is given by the Euler equations, which can be written as

$$\partial\mathcal{F}/\partial y_j = \frac{d}{dt}\left(\partial\mathcal{F}/\partial\dot{y}_j\right)$$

for state variable y_j. In light of eq. (B.1) we have

$$\partial\mathcal{F}/\partial K = e^{-\rho t}U'(C)\Big(\alpha(1-\beta)\chi^{\beta} - \delta_K\Big), \tag{B.2}$$

$$\partial\mathcal{F}/\partial\dot{K} = -e^{-\rho t}U'(C), \tag{B.3}$$

$$\partial\mathcal{F}/\partial X = e^{-\rho t}\frac{U'(C)}{G(X)}\Bigg(\alpha\beta(\chi^{\beta-1}G(X) + \delta_0\epsilon_G\Big(\frac{\dot{X}}{X}+1\Big) - \delta_0\Bigg), \tag{B.4}$$

$$\partial\mathcal{F}/\partial\dot{X} = -e^{-\rho t}\frac{U'(C)}{G(X)}, \tag{B.5}$$

where we have used $\chi = X/K$. Accordingly the Euler equations are

$$-\frac{CU''(C)}{U'(C)}\frac{\dot{C}}{C} = (\beta - 1)\chi^{\beta} - (\rho + \delta_K) \qquad \text{for K, and} \tag{B.6}$$

$$-\frac{CU''(C)}{U'(C)}\frac{\dot{C}}{C} = \alpha\beta\chi^{\beta-1}G(X) + \big(\delta_0\epsilon_G - 1\big)\Big(\frac{\dot{X}}{X} + 1\Big) - \rho \qquad \text{for X.} \tag{B.7}$$

Noting that $-\frac{CU''(C)}{U'(C)} = \theta$ (from **A4**) then gives eqs. (34)–(35) in the text.

The time at which society transitions from phase 1 to phase 2 can be determined in an analogous manner to that used in Appendix A. It must also be true that the state variables X and K move continuously between the two phases. That is, the starting values for X and K at the beginning of phase 2 are the terminal values at the end of phase 1.

The steady state of this system is defined by setting the right-hand sides of the Euler equations equal to zero, while imposing $\dot{X} = 0$:

$$\chi * = \left(\frac{\rho + \delta_K}{\beta - 1}\right)^{\frac{1}{\beta}}; \tag{B.8}$$

$$\alpha\beta(\chi^*)^{\beta-1}G(X^*)\delta_0\epsilon_G = 1 + \rho. \tag{B.9}$$

These two conditions then determine X^* and K^*. In addition, we note that

$$R^* = \frac{\delta_0 X^*}{G(X^*)},$$

$$I^* = \delta_K K^*,$$

$$C^* = F(X^*, K^*) - I^* - R^*.$$

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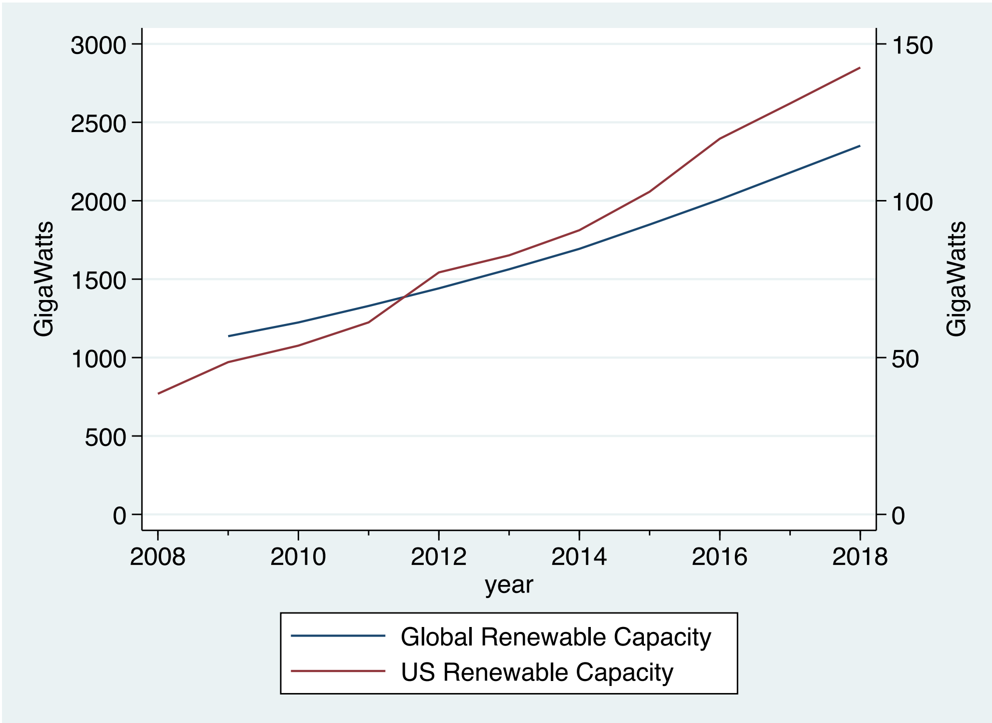

FIGURE 1. Renewable capacity, US and Global: 2008 – 2018

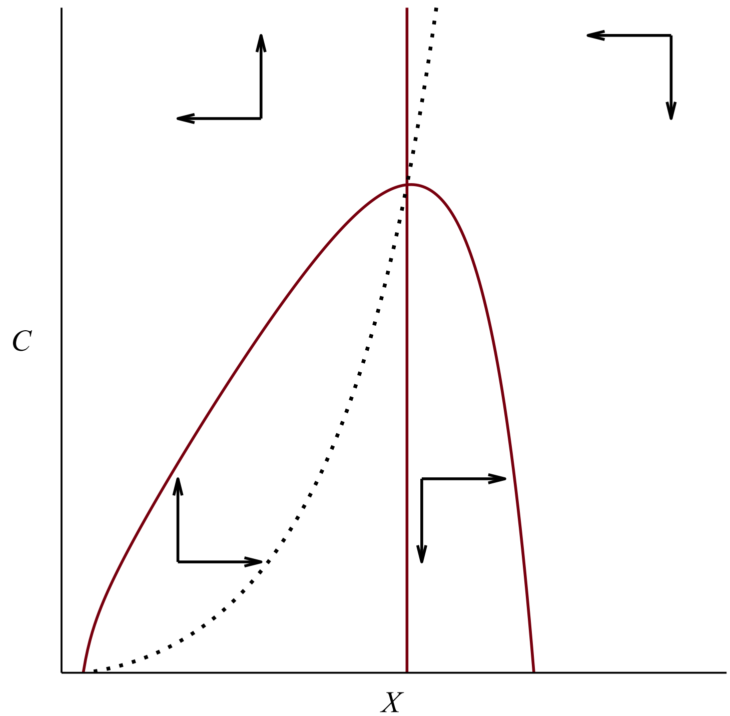

Figure 2. Stable paths

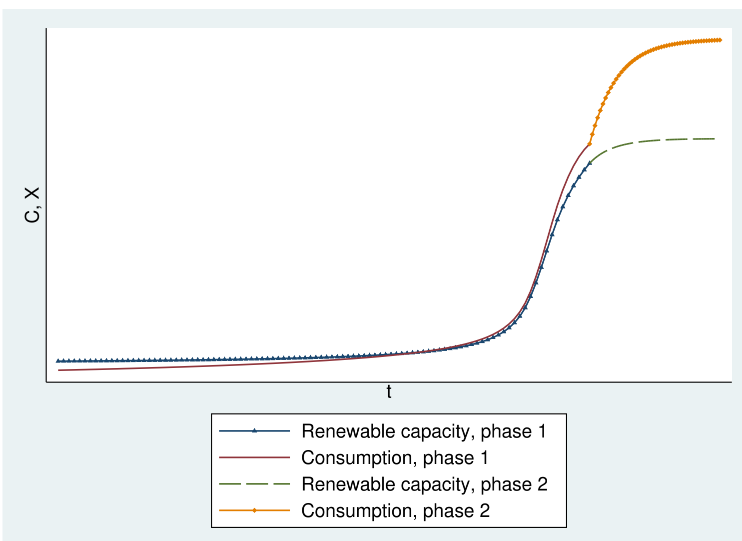

Figure 3. Time paths for renewable capacity and consumption, both phases

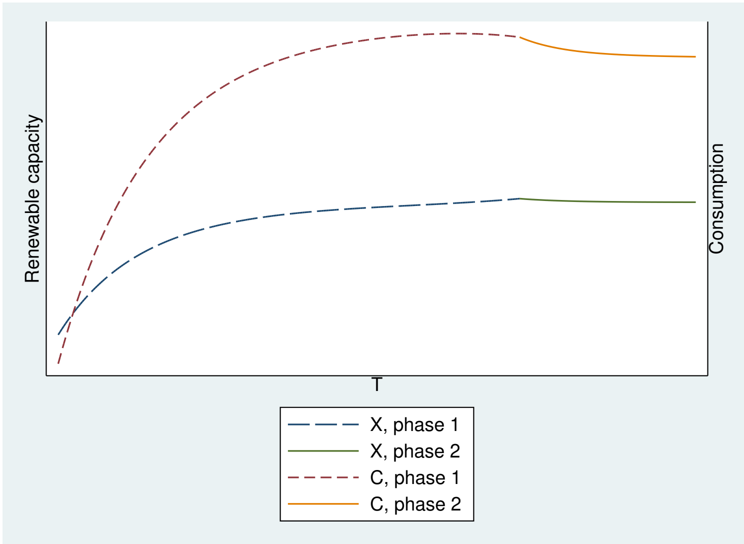

FIGURE 4. Time paths in "too much oil" case

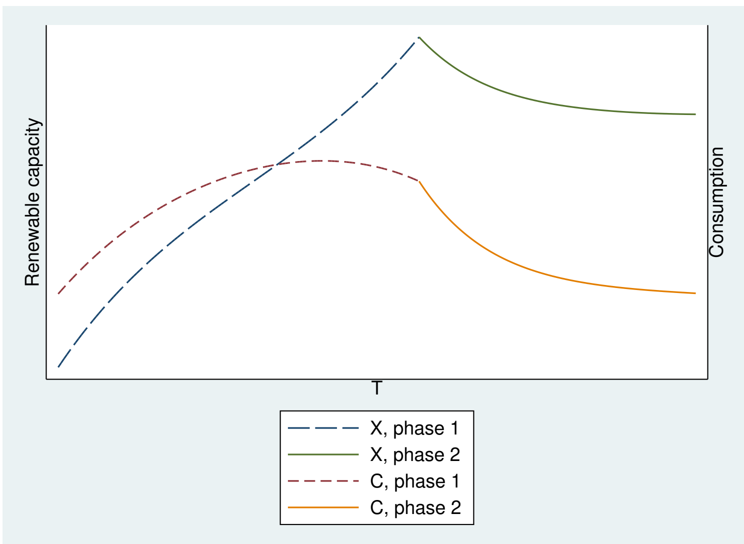

FIGURE 5. Time paths in "too much oil" case, closeup

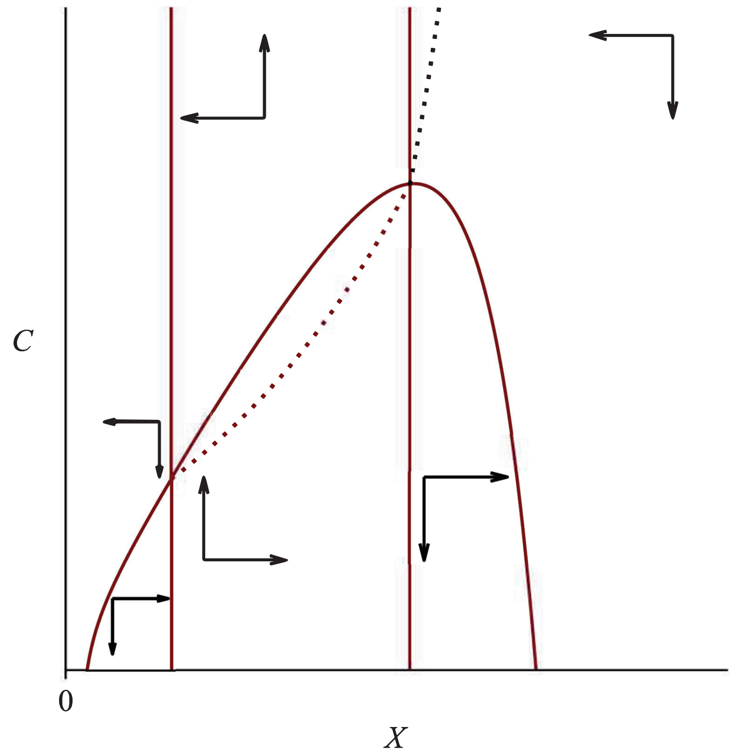

FIGURE 6. Two steady states with the stable paths

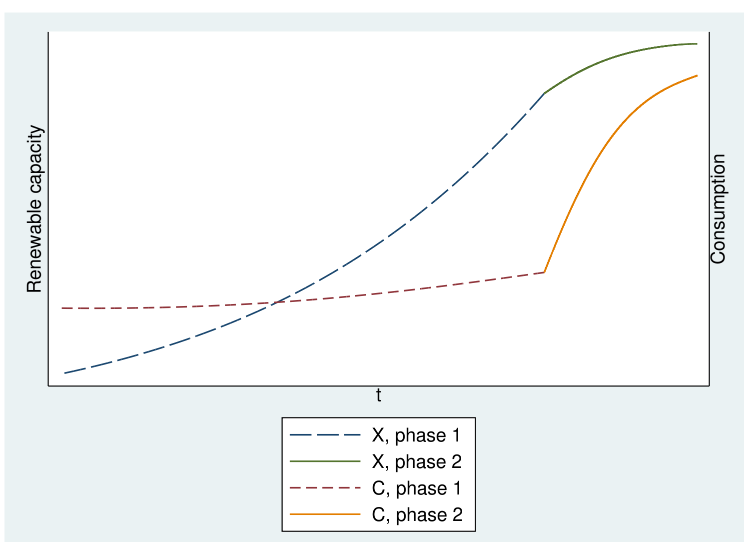

FIGURE 7. Time paths in two steady states case

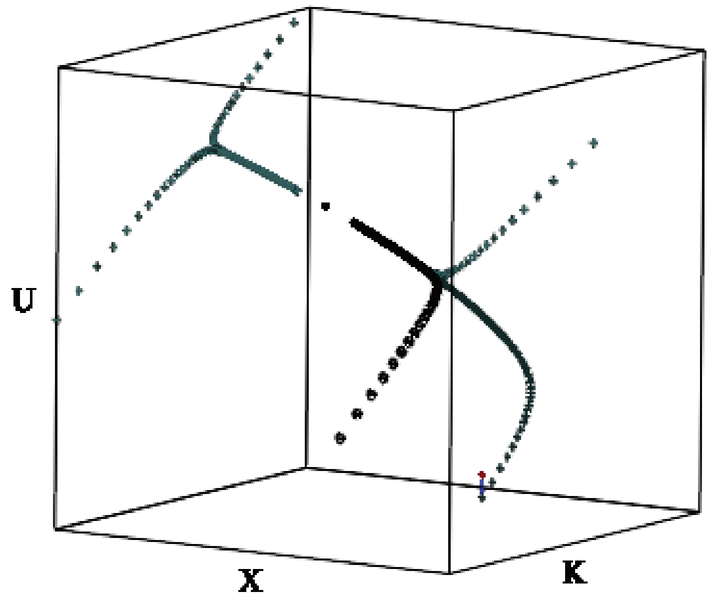

FIGURE 8. Stable paths, 2 input case